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Essays in microeconomics: information and learning

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ESSAYS IN MICROECONOMICS: INFORMATION AND LEARNING

by

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A thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
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CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

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has been approved by the Examining Committee for the thesis requirement for the Doctor of Philosophy degree in Economics at the May 2017 graduation.

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ABSTRACT

This dissertation contributes to the understanding of dynamic games in frictional markets. Specifically, it focuses on how information and search frictions influence outcomes in areas such as housing, over-the-counter markets, and online sales. I investigate how agents confront these frictions through learning, searching, and bargaining strategies that affect price formation and the allocation of resources.

In Chapter 1, we analyze a dynamic trading model of adverse selection where a seller can increase the frequency of strategic price quotes. A low-quality seller benefits more from trade and, therefore, searches more intensively than a high-quality seller. This makes a seller's contact carry negative information but a seller's availability become a stronger indicator of high quality. In the stationary environment, the two effects exactly offset each other, and reducing search costs is weakly beneficial to the seller. In the non-stationary environment, the relative strengths of the two effects vary over time, and reducing search costs can be detrimental to the seller.

In Chapter 2, I study a monopolistic pricing problem in which the consumer performs product research to determine whether or not to purchase the good. The consumer receives a signal of quality via a Brownian motion process with a type-dependent drift. I fully characterize the consumer's optimal strategy; she buys the product when she is sufficiently optimistic about the quality and ceases to pay for the signal when she is sufficiently pessimistic. I examine the implications of this behavior for the seller's optimal pricing decision. I find that the seller prefers to encourage product research when quality is likely

to be high and prefers to discourage research when quality is likely to be low. I show that a decrease in search costs or an increase in the quality of information can either raise or lower equilibrium price. I also extend the model so that the seller chooses both price and the level of quality dispersion and demonstrate that the optimal level of dispersion need not be extremal.

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TABLE OF CONTENTS

LIST OF FIGURES

Figure

CHAPTER 1
COSTLY SEARCH WITH ADVERSE SELECTION: SOLICITATION CURSE VS. ACCELERATION BLESSING

1.1 Introduction

We study a dynamic trading model in which a seller, with an indivisible object to sell, receives strategic price quotes from a sequence of randomly arriving buyers. We introduce two key features into this canonical trading environment. First, as in Akerlof (1970), the seller has private information about the quality of the object, where a high-quality unit is more valuable to both the seller and buyers than a low-quality unit. Second, the seller chooses the frequency of price quotes (i.e., the arrival rate of buyers) at an increasing cost. The cost can be interpreted as advertising expenditure (as in, e.g., Butters, 1977; Grossman and Shaprio, 1984) or search effort (as in, e.g., Burdett and Judd, 1983; Mortensen, 1986).

Our goal is to understand the joint effects of adverse selection and endogenous search (advertising) intensity in dynamic trading environments. A low-quality seller, due to her lower reservation value, enjoys more trade surplus and, therefore, has a stronger incentive to speed up trade than a high-quality seller. This affects buyers' inferences regarding the seller's underlying type and, therefore, their trading behavior, which in turn influences the seller's trading and search behavior. We formalize such inference problems of buyers and investigate equilibrium implications for market outcomes.[1]

[1]Our investigation is related to the literature on uninformative advertisements, which explains how they can be used to signal product quality. Signaling plays no role in our model because the seller has only one unit to sell (i.e., no repeat purchases) and her search intensity is not observable by buyers (i.e., no credible signal).

We identify the following two opposing effects, both of which stem from the fact that a low-quality seller chooses a higher search intensity than a high-quality seller. First, a seller who successfully finds a buyer is more likely to possess a low-quality unit. In other words, the very fact that a buyer has met a seller conveys bad news about the seller's type.[2] Following Lauermann and Wolinsky (2013), we call this effect the "solicitation curse." Second, a seller who has not traded yet is more likely to possess a high-quality unit. In other words, the fact that a unit is still available is good news about its quality. Note that even without endogenous intensity, a low-quality seller trades faster than a high-quality seller, because of the difference in reservation prices. Endogenous intensity makes a low-quality seller trade even faster than a high-quality seller. For this reason, we call this effect the "acceleration blessing."

To be formal, refer to the probability that the seller who remains in the game is the high type as buyers' *unconditional* beliefs, and the corresponding probability when a buyer actually faces the seller as buyers' *conditional* beliefs. These two values are identical if both seller types have the same search intensity. In our model, the low-type seller chooses a higher search intensity than the high-type seller, and thus buyers' conditional beliefs fall short of their unconditional beliefs. The solicitation curse refers to this downward adjustment from buyers' unconditional beliefs to conditional beliefs. Meanwhile, the difference in search intensities drives up buyers' unconditional beliefs beyond the level that is induced only by the difference in reservation prices. This additional increase of buyers'

[2]It is noteworthy that, although we obtain this effect in a fully rational framework, there is both experimental and empirical evidence about this phenomenon. See, e.g., Kirmani (1990, 1997), Kirmani and Wright (1989), and Kwoka (1984).

unconditional beliefs is the acceleration blessing. We study how these two effects manifest themselves and interact with each other in both stationary and non-stationary environments.

In Section 1.3, we consider an opaque trading environment where buyers do not receive any information about the seller's trading history. In this case, all buyers have the same beliefs about the seller's type and, therefore, play identical offer strategies. The environment is stationary from the seller's viewpoint and, therefore, each seller type's optimal search intensity is time-invariant. In this case, the aforementioned two effects take simple forms. We quantify the two effects and show that their magnitudes are identical for any strategy profile. In other words, environmental stationarity implies that the two effects exactly offset each other, and the difference in search intensities does not directly affect buyers' conditional beliefs. Nevertheless, endogenous search intensity still influences the players' equilibrium strategies. We provide a full equilibrium characterization and explain the effects of endogenous search intensity on market outcomes.

In Section 1.4, we examine a non-stationary version of the model. Specifically, we consider the case in which buyers observe the seller's time-on-the-market (i.e., how long the seller has been on the market). The observability assumption allows us to study how the seller's optimal search intensity and buyers' unconditional and conditional beliefs evolve over time. In this non-stationary model, the players' strategies and beliefs depend on the seller's time-on-the-market, and thus the solicitation curse and the acceleration blessing take more complex forms. The acceleration blessing induces buyers' (both unconditional and conditional) beliefs to converge to 1, which is in stark contrast to a common result in the literature that buyers' beliefs stay bounded away from 1. As in the stationary model, the solicitation curse brings down buyers' conditional beliefs relative to their unconditional beliefs. Unlike in the stationary model, its strength relative to the acceleration blessing changes over time. In particular, it outweighs the acceleration blessing for a certain length of time, which leads to non-monotonicity of buyers' conditional beliefs. We show that, although buyers' unconditional beliefs monotonically increase over time, their conditional beliefs first increase, then weakly decrease, and finally increase and converge to 1.

In both environments, we examine the welfare effects of lowering search costs. It certainly has a direct benefit to the seller. However, there is an opposing indirect effect, which comes from the fact that a change in search costs has unequal effects for different sellers. A low-quality seller is more sensitive to a decrease in search costs and increases her search intensity more than a high-quality seller. This means that the solicitation curse worsens and, therefore, buyers become even more cautious about offering a high price, which negatively affects the seller. In the stationary environment, this strategic effect cannot overcome the direct effect: the low-type seller's expected payoff either increases or stays constant, the latter being the case if and only if search costs are sufficiently small. In the non-stationary environment, however, the strategic effect can dominate, and thus lowering search costs can be harmful to the seller. We provide a sufficient condition under which the seller is worse off with lower search costs and also explain how buyers can benefit from the seller's lower search costs.

Adverse selection has been studied in various dynamic environments. Janssen and Roy (2002, 2004) study an infinitely repeated competitive market in which there is a single clearing price in each period, whereas Hendel and Lizzeri (1999, 2002) and Hendel, Lizzeri and Siniscalchi (2005) consider a competitive environment in which units are classified according to their vintages. Adverse selection has also been analyzed in non-stationary competitive environments (e.g., Daley and Green, 2012; Fuchs and Skrzypacz, 2013a,b; Fuchs, Öry and Skrzypacz, 2015) as well as in the bilateral bargaining context (e.g., Evans, 1989; Vincent, 1989; Deneckere and Liang, 2006). Competitive and bilateral bargaining environments do not feature search frictions and, therefore, cannot be used to address our economic questions regarding endogenous search (advertising) intensity.

Our article is more closely related to a growing literature on adverse selection in sequential (random) search environments. This literature has two strands, one studying a market environment (e.g., Moreno and Wooders, 2010, 2016; Camargo and Lester, 2014; Chiu and Koeppl, 2014) and the other, as in this article, focusing on a single searcher's problem (e.g., Hörner and Vieille, 2009; Zhu, 2012; Lauermann and Wolinsky, 2016). The specific model is based on the tractable continuous-time framework by Mortensen (1986) and Kim (2015). The former explains the effects of endogenous search intensity in the canonical sequential search framework, whereas the latter introduces adverse selection into the framework. Our model can be interpreted as one that extends Mortensen (1986) to incorporate adverse selection, or one that extends Kim (2015) to allow for endogenous search intensity.[3] To our knowledge, we are the first to study the joint effects of adverse selection and endogenous search intensity in the sequential search framework.

[3]The continuous-time framework in this article has proved to be useful in other contexts. See, e.g., Kaya and Kim (2015) and Hwang (2015). It is worth noting that we provide another extension of Kim's model in which the seller's search intensity is exogenously given but can depend on her type.

Adverse selection also has been incorporated into the directed search framework (e.g., Inderst and Müller, 2002; Guerrieri, Shimer and Wright, 2010; Chang, 2014; Guerrieri and Shimer, 2014).[4] Directed search provides an alternative way to endogenize search intensity in the presence of adverse selection: in directed search models, each agent effectively chooses a pair of transaction price and search intensity. The main economic effects are, however, distinct from those in this article. In directed search models, the main working mechanism is sorting among different seller types: a low-quality seller prefers a submarket with a lower price but a higher search intensity, whereas a high-quality seller prefers a submarket with a higher price but a lower search intensity. The central question is when and how such sorting occurs. Indeed, it has been shown that a fully separating equilibrium is the unique equilibrium in most models.[5] Therefore, in equilibrium, buyers' inference problems become trivial, but they are the central issue in our analysis.

Our article is closely related to Lauermann and Wolinsky (2013). In a nutshell, they study a simultaneous-search counterpart to our model: in their model, the seller chooses the number of buyers who will simultaneously offer her a price. They also identify a solicitation effect, which, as in this article, stems from the fact that different seller types have different incentives to increase the number of buyers.[6] However, an acceleration effect is

[4]A precursor to this literature is a seminal article by Wilson (1980), who studies a static competitive market with three different trading conventions, which differ in terms of who sets the price among an auctioneer, buyers, and sellers.

[5]Chang (2014) shows that a fully separating equilibrium may not exist if sellers differ not only in terms of the quality of their units, but also in terms of their private values. She proves that a semi-pooling equilibrium exists in such an environment.

[6]One difference is the source of seller heterogeneity. In our article, different seller types have different reservation values, whereas in Lauermann and Wolinsky (2013), different seller types have

absent in their model, because it is a static environment. In addition, their main economic question is substantially different from ours: they seek the condition on the signal generating process that guarantees information aggregation (meaning that the winning price coincides with the unit's value to buyers), whereas our main question is the impact of endogenous search intensity on trading outcomes (dynamics).

Finally, there are two articles that report a similar result to our main comparative statics result (that reducing search costs can be detrimental to the seller). Grossman and Shaprio (1984) show that increasing advertising costs can increase firms' profits in a model of informative advertisements under monopolistic competition. Fuchs and Skrzypacz (2013b) show that decreasing the frequency of trade (by closing the market for a certain length of time) can be welfare-improving in a dynamic model with continuous trade (i.e., no search frictions). The mechanisms behind these results, however, are fundamentally different from the one in this article. The result is driven by firms' optimal price responses in Grossman and Shaprio (1984) (namely that firms can set higher prices when an increase in advertising costs reduces competition), whereas it is because closing the market decreases the seller's incentive to delay trade in Fuchs and Skrzypacz (2013b).

The remainder of the article is organized as follows. We introduce the environment in Section 1.2. We study an opaque (therefore, stationary) model in Section 1.3 and a nonstationary model in Section 1.4. We conclude in Section 1.5 by providing a few relevant discussions.

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an identical reservation value but buyers receive informative signals about the seller's type.

1.2 Environment

Sequential Search.

We consider a canonical sequential search environment in continuous time. A seller wishes to sell an indivisible object and sequentially meets buyers. Upon arrival each buyer offers a price, and the seller decides whether to accept the price or not. If an offer is accepted, then the seller and the buyer trade and the game ends. Otherwise, the buyer leaves and the seller waits for the next buyer.

Adverse Selection.

The good is either of high quality (H) or of low quality (L). If the good is of quality $a = H, L$, then it yields flow utility rc_a to the seller and flow utility rv_a to buyers, where r is the common discount rate. Notice that c_a and v_a represent the stock values of a type-a unit for the seller and for buyers, respectively, because $x = \int_0^\infty e^{-rt} rx dt$. A high-quality unit is more valuable to both the seller and buyers (i.e., $c_L < c_H$ and $v_L < v_H$). There are always gains from trade (i.e., $c_a < v_a$ for each $a = H, L$), but the quality of the good is private information to the seller. It is common knowledge that buyers assign probability $\widehat{q}$ to the event that the seller's good is of high quality at the beginning of the game. All agents are risk neutral. If a buyer's offer p is accepted by the type-a seller, then the buyer's utility is $v_a - p$, whereas the seller's (net) utility is $p - c_a$, at the time of exchange.

We focus on the case where adverse selection is severe enough to impede socially desirable trade. Formally, we maintain the following assumption, which is common in the adverse selection literature:

Assumption 1.1. *(Severe Adverse Selection)*

$$\widehat{q}v_H + (1-\widehat{q})v_L < c_H.$$

The left-hand side is the unconditional expected value of the good to buyers, whereas the right-hand side is the high-type seller's reservation value (lowest acceptable price). This assumption guarantees that it cannot be an equilibrium that both seller types trade with the first buyer. In other words, delay is unavoidable.

Search (Advertising) Technology.

The seller can increase the arrival rate of buyers. As signaling through the choice of search intensity is not the subject of this article, we assume throughout that the seller's choice of search intensity is not observable to buyers. The search technology is represented by a function $\phi : [\underline{\lambda}, \infty) \to [0, \infty)$ where $\phi(\lambda)$ denotes the flow search cost the seller must incur in order to obtain search intensity λ. In other words, if the seller pays constant flow search cost $\phi(\lambda)$, then buyers arrive according to a Poisson process of rate λ. In Section 1.3 (stationary model), we impose only standard restrictions on the cost function $\phi(\cdot)$: it is strictly increasing and strictly convex (i.e., $\phi'(\lambda), \phi''(\lambda) > 0$), $\phi(\underline{\lambda}) = 0$, and $\lim_{\lambda \to \underline{\lambda}} \phi'(\lambda) = 0$. In Section 1.4 (non-stationary model), we focus on a simple binary case: the seller can increase her search intensity only from $\underline{\lambda}$ to $\overline{\lambda}(> \underline{\lambda})$ at cost $\phi(\equiv \phi(\overline{\lambda}))$. Equilibrium characterization of the non-stationary model with general search technology is quite involved and relegated to the online appendix. Finally, to avoid triviality, we assume that $\underline{\lambda} > 0$. This can be interpreted as the baseline search intensity which the seller obtains for free. The role of this assumption will be clear in the equilibrium analysis. It will also be clear that, although we require $\underline{\lambda} > 0$, $\underline{\lambda}$ can take any small value.

1.3 Stationary Model

In this section, we consider an opaque search environment in which buyers do not observe any part of the seller's trading history. The problem is *stationary* from the seller's viewpoint, because all buyers have common beliefs and, therefore, play an identical offer strategy. We focus on the equilibrium in which the seller adopts a stationary acceptance strategy. As is common in sequential search problems, the seller's optimal acceptance strategy is described by a reservation price: each seller type accepts a price if it is weakly above her reservation price and rejects if it is below.

For each $a = H, L$, we let p_a and λ_a denote the type-a seller's reservation price and search intensity, respectively. In addition, we represent buyers' offer strategies as the right-continuous distribution function F, where $F(p)$ denotes the probability that each buyer offers a price weakly below p. For notational simplicity, we let $F_{-}(p)$ denote the left limit of the distribution function F at p (i.e., $F_{-}(p) \equiv \lim_{x \to p-} F(x)$).

1.3.1 Buyers' Beliefs

We first analyze how buyers' beliefs are determined in the stationary environment. This allows us to understand how to identify the solicitation effect and the acceleration effect and how they interact with each other.

In the stationary model, buyers face two types of uncertainty, one about the seller's type and the other about their position in the sequence of buyers (which Zhu (2012) refers to as "contact uncertainty"). The combination of these two gives rise to a non-trivial inference problem. In particular, buyers' beliefs about the seller's type may not coincide with their prior beliefs $\widehat{q}$. This is because different seller types leave the game at different rates, and thus the probability of the high type changes over time. If buyers could observe the seller's trading history, then their beliefs would begin with $\widehat{q}$ and be updated through Bayes' rule for all subsequent points in time. In the current environment, however, buyers receive no information about the seller's trading history and, therefore, contact uncertainty must be taken into account in determining their beliefs.

Let q^u represent the probability that a seller who is still playing the game (i.e., has not traded yet) is the high type. In other words, q^u denotes buyers' unconditional beliefs that the seller is the high type. To determine q^u, observe that, because each seller type accepts any price weakly above her reservation price and the high type (low type) meets buyers at rate λ_H (λ_L), the probability that the high-type seller does not trade until time t is equal to $e^{-\lambda_H(1-F_-(p_H))t}$, whereas the corresponding probability for the low-type seller is equal to $e^{-\lambda_L(1-F_-(p_L))t}$. Combining this with buyers' prior beliefs $\widehat{q}$ leads to

$$q^u = \frac{\widehat{q}\int_0^\infty e^{-\lambda_H(1-F_-(p_H))t}dt}{\widehat{q}\int_0^\infty e^{-\lambda_H(1-F_-(p_H))t}dt + (1-\widehat{q})\int_0^\infty e^{-\lambda_L(1-F_-(p_L))t}dt} = \frac{\frac{\widehat{q}}{\lambda_H(1-F_-(p_H))}}{\frac{\widehat{q}}{\lambda_H(1-F_-(p_H))} + \frac{1-\widehat{q}}{\lambda_L(1-F_-(p_L))}}. \tag{1.1}$$

We assume that $q^u = \widehat{q}$ if $F_-(p_H) = F_-(p_L) = 1$, $q^u = 1$ if $F_-(p_H) = 1 > F_-(p_L)$, and $q^u = 0$ if $F_-(p_H) < F_-(p_L) = 1$.[7] Intuitively, if $F_-(p_H) = F_-(p_L) = 1$, then the seller never trades, and thus buyers' beliefs stay constant at their prior level $\widehat{q}$. If $F_-(p_a) = 1 > F_-(p_b)$, then the type-a seller never trades, whereas the type-b seller trades in finite

[7]There is no logical inconsistency between this assumption and the fact that a buyer can be the first buyer to the seller. In the limit as either $F_-(p_H)$ or $F_-(p_L)$ tends to 0, one seller type never trades and, therefore, meets infinitely many buyers. The probability that a buyer is the first buyer converges to 0, and q^u also approaches either 0 or 1.

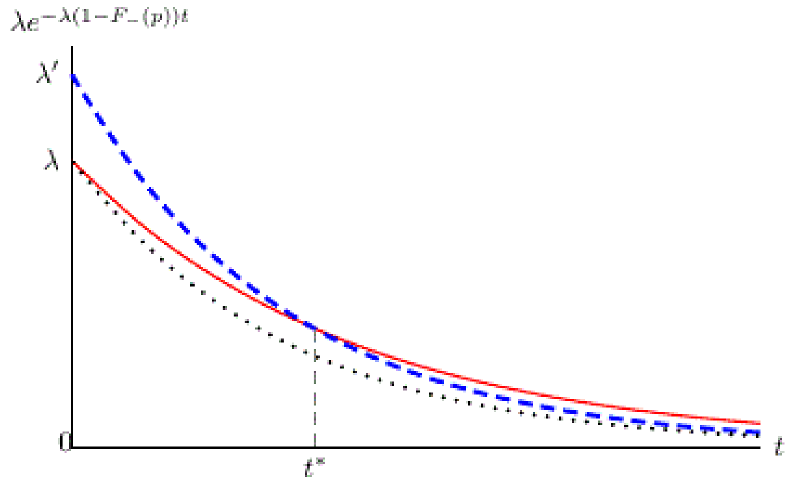
time with probability 1. Therefore, the probability that a seller remaining in the market is of type a is equal to 1.

Notice that q^u may differ from $\widehat{q}$ for two reasons. First, the two seller types have different reservation prices (i.e., $p_H \neq p_L$). This is familiar in the literature on dynamic adverse selection and has received significant attention because of its unique implication for trade efficiency, namely that in equilibrium, buyers' incentive constraints are sufficiently relaxed that all seller types eventually trade in stationary environments.[8] Second, the difference in search intensities ($\lambda_H \neq \lambda_L$) also contributes to the departure of q^u from $\widehat{q}$. If $\lambda_L > \lambda_H$, then this makes the low type leave the game relatively faster than the high type. The opposite holds if $\lambda_L < \lambda_H$. As shown shortly, in our model with endogenous search intensity, the second effect necessarily goes in the same direction as, and thus reinforces, the first effect. Therefore, we refer to this further difference in unconditional beliefs due to different search intensities as the acceleration effect.

The difference in search intensities has one more implication for buyers' inferences: buyers have different beliefs about the seller's type, depending on whether they actually face the seller or not. If $\lambda_a > \lambda_b$, then the type-a seller faces relatively more buyers than type b. This means that a buyer is more likely to face type a than type b and, therefore, should adjust his belief accordingly. To be formal, denote by q^c the probability that a buyer assigns to the event that the seller is the high type, *conditional* on the event that he has

[8] As clarified shortly, in equilibrium, $p_L < p_H$. This implies that the low-type seller leaves the market faster than the high-type seller and, therefore, the average quality perceived by buyers is higher than $\widehat{q}$. This is the underlying reason why the high-type seller, who cannot trade in a static environment, can trade in the current dynamic environment.

Figure 1.1: Search Intensity and Meeting Rate



The effects of varying the type-a seller's search intensity on the rate at which buyers meet the seller type conditional on the seller's time-on-the-market t. Each line depicts $\lambda e^{-\lambda(1-F_{-}(p))t}$ (solid), $\lambda e^{-\lambda'(1-F_{-}(p))t}$ (dotted), and $\lambda' e^{-\lambda'(1-F_{-}(p))t}$ (dashed), where $\lambda < \lambda'$.

actually met the seller. The relationship between q^u and q^c is given by

$$q^c = \frac{q^u \lambda_H}{q^u \lambda_H + (1-q^u)\lambda_L}. \tag{1.2}$$

Clearly, q^c is smaller (larger) than q^u if $\lambda_L > (<)\lambda_H$. We refer to this adjustment from q^u to q^c as the solicitation effect.

Combining (1.1) and (1.2) yields

$$q^c = \frac{q^u \lambda_H}{q^u \lambda_H + (1-q^u)\lambda_L} = \frac{\frac{\widehat{q}}{1-F_{-}(p_H)}}{\frac{\widehat{q}}{1-F_{-}(p_H)} + \frac{1-\widehat{q}}{1-F_{-}(p_L)}}. \tag{1.3}$$

Notice that the search intensity parameters, λ_L and λ_H, do not appear in this expression. This, however, does not mean that the difference in search intensities has no effect on market outcomes. As shown shortly, buyers' equilibrium offer strategies are influenced by

λ_L and λ_H, which are in turn determined in equilibrium. It only means that in stationary environments, the two effects, the acceleration effect and the solicitation effect, are of the same magnitude for all strategy profiles.

Figure 1.1 illustrates the underlying reason why the two effects offset each other. If a seller type's search intensity increases (from λ to λ'), then the type leaves the market faster, and thus the probability that the seller type stays in the market until time t decreases at each t (shift-down from the solid line to the dotted line). However, conditional on t, the seller meets relatively more buyers (shift-up from the dotted line to the dashed line). Overall, the probability that a buyer meets the seller type increases conditional on $t < t^*$ but decreases conditional on $t > t^*$. Our result indicates that in stationary environments where buyers do not observe the seller's time-on-the-market, there are no net gains. In other words, in Figure 1.1, integration over t yields an identical value for both the solid line and the dashed line.

1.3.2 Equilibrium Offer Strategies and Reservation Prices

We proceed to characterize buyers' equilibrium offer strategies F and both seller types' reservation prices p_H and p_L. We endogenize equilibrium search intensities λ_H and λ_L in the subsequent analysis.[9]

We make two preliminary observations. First, by the standard Diamond paradox logic, in equilibrium no buyer offers strictly above c_H.[10] It is then straightforward that $p_H =$

[9]As we take λ_H and λ_L as given, the analysis here can be interpreted as a full equilibrium characterization of the model with exogenous search intensity. This characterization generalizes the one in Kim (2015) where attention is restricted to the case where $\lambda_H = \lambda_L$.

[10]To be formal, let $\widehat{p}$ be the supremum price buyers offer after all histories in any equilibrium,

c_H and $p_L < c_H$: the latter result is due to the fact that the low-type seller's reservation price is maximized when buyers always offer c_H, but even then $p_L = (rc_L + \lambda_L c_H)/(r + \lambda_L) < c_H$. Second, in equilibrium no buyer has an incentive to offer strictly above c_H (dominated by c_H), or between c_H and p_L (dominated by p_L). Therefore, there is no loss of generality in assuming that each buyer offers either c_H, p_L, or a losing price (below p_L). For each $a = H, L$, we let σ_a denote the probability that each buyer offers the reservation price of the type-a seller.

A crucial observation is that in equilibrium buyers must offer both c_H and p_L with positive probabilities and, therefore,

$$q^c(v_H - c_H) + (1 - q^c)(v_L - c_H) = (1 - q^c)(v_L - p_L) \Leftrightarrow \frac{q^c}{1 - q^c} = \frac{c_H - p_L}{v_H - c_H}. \quad (1.4)$$

Suppose buyers never offer c_H (i.e., $\sigma_H = 0$). In this case, the low-type seller's reservation price p_L shrinks to c_L, because she never obtains strictly more than her reservation price and, therefore, her expected payoff is the same as when she does not trade at all. This implies that buyers would offer only $p_L = c_L$ (i.e., $\sigma_L = 1$), which in turn implies $q^c = 1$ (see equation (1.3)). If so, however, a buyer strictly prefers offering c_H (which yields expected payoff $v_H - c_H$) to $p_L = c_L$ (which yields payoff 0), which is a contradiction. Now suppose buyers never offer p_L (i.e., $\sigma_L = 0$). In this case, by equation (1.3), $q^c = \widehat{q}$. Assumption 1.1 implies that $q^c(v_H - c_H) + (1 - q^c)(v_L - c_H) < 0$, and thus $\sigma_H = 0$ as well. But then, the same contradiction as for the previous case arises.

---

and suppose $\widehat{p} > c_H$. Define $\widehat{p}' \equiv \max_{a=H,L}(rc_a + \lambda_a \widehat{p})/(r + \lambda_a)$, so that $\widehat{p}'$ denotes the seller's maximal reservation price. As $c_H < \widehat{p}$, $\widehat{p}' < \widehat{p}$. A contradiction arises, if this inequality is combined with the fact that buyers never offer more than $\widehat{p}'$.

The following proposition fully describes the unique equilibrium strategy profile that corresponds to each pair of λ_H and λ_L.

Proposition 1.1. *Given λ_H and λ_L, the unique equilibrium is given as follows: if*

$$\frac{r(v_L - c_L)}{\lambda_L(v_H - c_H)} \leq \frac{\widehat{q}}{1-\widehat{q}}, \tag{1.5}$$

then

$$p_L = v_L,\ q^c = \frac{c_H - v_L}{v_H - v_L},\ \sigma_H = \frac{r}{\lambda_L}\frac{v_L - c_L}{c_H - v_L},\ \textit{and}\ \sigma_L = \sigma_H\left(\frac{1-\widehat{q}}{\widehat{q}}\frac{c_H - v_L}{v_H - c_H} - 1\right).$$

Otherwise,

$$p_L = c_L + \frac{\widehat{q}}{1-\widehat{q}}\frac{\lambda_L}{r}(v_H - c_H),\ q^c = \frac{c_H - p_L}{v_H - p_L},\ \sigma_H = \frac{\widehat{q}}{1-\widehat{q}}\frac{v_H - c_H}{c_H - p_L},\ \textit{and}\ \sigma_L = 1 - \sigma_H.$$

Proof. See Appendix A. □

An increase in λ_L reduces the low-type seller's cost to wait for c_H and, therefore, increases her reservation price p_L. However, p_L is accepted only by the low type and, therefore, cannot exceed v_L. Equilibrium takes one of the following two structures: $p_L < v_L$ if λ_L is small (precisely, condition (1.5) is violated), whereas $p_L = v_L$ if λ_L is large (condition (1.5) holds). In the former case, buyers obtain a strictly positive expected payoff and, therefore, never make a losing offer (i.e., $\sigma_H + \sigma_L = 1$). In the latter case, buyers make zero expected payoff. Losing offers are necessary in equilibrium to provide appropriate incentives for both the low-type seller ($p_L = v_L$) and buyers ($q^c(v_H - c_H) + (1 - q^c)(v_L - c_L) = 0$).

Notice that λ_H does not play any role in Proposition 1.1. There are two reasons for this. First, as explained above, buyers' conditional beliefs are independent of λ_H and λ_L.

Second, the high-type seller always obtains zero expected payoff, and thus her incentive constraint never binds. This implies that equilibrium prices and beliefs depend only on the low-type seller's incentives (thus, on λ_L, but not on λ_H).

1.3.3 Endogenizing Search Intensity

We complete equilibrium characterization by endogenizing each seller type's search intensity. In other words, we look for a pair of λ_H and λ_L such that for both $a = H, L$, the type-a seller has an incentive to choose λ_a given search technology $\phi(\cdot)$ and the corresponding equilibrium strategy profile as characterized in Proposition 1.1.

One straightforward result is that the high-type seller has no incentive to increase her search intensity and, therefore, chooses $\underline{\lambda}$.[11] A seller's incentive to speed up trade stems from her desire to enjoy trade surplus as soon as possible. However, buyers never offer strictly above c_H, and thus the high-type seller receives zero expected payoff and has no incentive to incur search costs. To the contrary, the low-type seller receives positive trade surplus and, therefore, chooses $\lambda_L > \underline{\lambda}$.

The low-type seller obtains a strictly positive payoff only when she trades at c_H. Therefore, her reservation price (equivalently, her continuation payoff) depends only on σ_H. The relevant continuous-time Bellman equation is given as follows:

$$p_L = \max_{\lambda \geq \underline{\lambda}} \{-\phi(\lambda)dt + rc_L dt + \lambda\sigma_H dt \cdot c_H + e^{-(r+\lambda\sigma_H)dt} p_L\}.$$

The equation reflects the fact that if the low-type seller chooses search intensity λ_L, then

[11] This significantly simplifies the analysis, but is not essential for the main insights in this article. In an earlier version, we considered an alternative bargaining protocol with which the high-type seller also obtains a strictly positive expected payoff and showed that most results in the baseline model go through unchanged.

her flow payoff is equal to $-\phi(\lambda_L) + rc_L$, she receives offer c_H at rate $\lambda_L\sigma_H$, and her reservation price stays constant over time. Rewriting the equation in flow terms,[12]

$$r(p_L - c_L) = \max_{\lambda \geq \underline{\lambda}}\{-\phi(\lambda) + \lambda\sigma_H(c_H - p_L)\}. \quad (1.6)$$

Intuitively, the net flow value of holding a low-quality unit comes from the possibility of selling it at c_H, in which case the stock value increases by $c_H - p_L$, minus the flow search cost.

Equation (1.6) leads to the following two equilibrium conditions. First, because the optimal search intensity λ_L must maximize the right-hand side,

$$\phi'(\lambda_L) = \sigma_H(c_H - p_L). \quad (1.7)$$

The strict convexity of $\phi(\cdot)$ ensures the uniqueness of λ_L. Second, the optimal search intensity λ_L must indeed satisfy equation (1.6). Therefore,

$$r(p_L - c_L) = -\phi(\lambda_L) + \lambda_L\sigma_H(c_H - p_L). \quad (1.8)$$

It is clear that both λ_L and p_L are strictly increasing in σ_H: the more frequently buyers offer c_H, the higher expected payoff the low-type seller obtains and the more intensively she searches.

Combining equations (1.7) and (1.8) with Proposition 1.1 leads to full equilibrium characterization, which is summarized in the following proposition. We present only the necessary results for p_L and λ_L, as all other equilibrium variables can be easily recovered from Proposition 1.1.

[12] The flow equation can be derived from the previous (stock) equation as follows: subtract $e^{-rdt}p_L$ and divide both sides by dt. Arranging the terms with the fact that dt is arbitrarily close to 0 and $\lim_{x\to 0}(1 - e^{-ax})/x = a$ for any $a \in \mathcal{R}$ yields equation (1.6).

Proposition 1.2. *In the stationary model, there exists a unique equilibrium. Let $\tilde{\lambda}$ be the unique value such that $r(v_L - c_L) = \tilde{\lambda}\phi'(\tilde{\lambda}) - \phi(\tilde{\lambda})$. If*

$$\frac{r(v_L - c_L) + \phi(\tilde{\lambda})}{\tilde{\lambda}(v_H - c_H)} \leq \frac{\widehat{q}}{1 - \widehat{q}}, \tag{1.9}$$

then $p_L = v_L$ and $\lambda_L = \tilde{\lambda}$. Otherwise, $p_L < v_L$ and $\lambda_L = (\phi')^{-1}\left(\widehat{q}(v_H - c_H)/(1 - \widehat{q})\right) < \tilde{\lambda}$.

Proof. See Appendix A. □

Condition (1.9) is analogous to condition (1.5) in Proposition 1.1. To understand the condition more clearly, consider a parametric example where $\phi(\lambda) = b(\lambda - \underline{\lambda})^2$ for some $b > 0$. In this case,

$$\tilde{\lambda}^2 - \underline{\lambda}^2 = \frac{r(v_L - c_L)}{b},$$

and condition (1.9) shrinks to

$$\frac{2r(v_L - c_L)}{(\tilde{\lambda} + \underline{\lambda})(v_H - c_H)} \leq \frac{\widehat{q}}{1 - \widehat{q}}. \tag{1.10}$$

The inequality holds if and only if $\underline{\lambda}$ is sufficiently large or b is sufficiently small. Both of these can be interpreted as search frictions being small: in the former case, the seller meets buyers quickly even without incurring any search cost, whereas in the latter case, it is not so costly to increase search intensity. In either case, $\lambda_L = \tilde{\lambda}$ is sufficiently large that condition (1.9) holds.

We return to the determination of buyers' beliefs and discuss the effects of endogenous search intensity on them. Proposition 1.2 states that in equilibrium $p_L < p_H = c_H$

and $\lambda_L > \lambda_H = \underline{\lambda}$. Applying these to equations (1.1) and (1.3) yields the following unconditional and conditional beliefs:

$$\frac{q^u}{1-q^u} = \frac{\widehat{q}}{1-\widehat{q}} \frac{\lambda_L}{\underline{\lambda}} \frac{\sigma_H + \sigma_L}{\sigma_L}, \text{ and } \frac{q^c}{1-q^c} = \frac{\widehat{q}}{1-\widehat{q}} \frac{\sigma_H + \sigma_L}{\sigma_L}.$$

Buyers' unconditional beliefs q^u exceed their prior beliefs $\widehat{q}$ for two reasons. The first one, reflected in $(\sigma_H + \sigma_L)/\sigma_L$, is familiar in the literature. The high-type seller has a higher reservation price and, therefore, stays longer than the low-type seller. This increases the probability of the high type beyond the initial probability $\widehat{q}$. The second one, reflected in $\lambda_L/\underline{\lambda}$, is the acceleration effect. The low-type seller chooses a higher search intensity and, therefore, leaves the market even faster than the high-type seller. This effect further relaxes buyers' incentive constraints to offer c_H (thus, the acceleration blessing).

Buyers' conditional beliefs q^c exceed their prior beliefs $\widehat{q}$ but fall short of their unconditional beliefs q^u. This is because the solicitation effect is negative (thus, the solicitation curse). The low-type seller searches more intensively than the high-type seller. Therefore, a buyer should adjust down his belief about the seller's type once he actually faces (is solicited by) the seller. As explained above, in stationary environments, the acceleration effect and the solicitation effect are of the same magnitude, and thus buyers' conditional beliefs are independent of λ_L and $\lambda_H = \underline{\lambda}$.

1.3.4 Effects of Reducing Search Costs

We now study the effects of reducing search costs on the players' expected payoffs. As it is not clear how to measure a change of the function $\phi(\cdot)$, we restrict attention to the parametric case where $\phi(\lambda) = b(\lambda - \underline{\lambda})^2$. In this case, a decrease in b can be naturally interpreted as a decrease in search costs.

The following result is immediate from the closed-form solution for the parametric case.

Corollary 1.1. *Suppose* $\phi(\lambda) = b(\lambda - \underline{\lambda})^2$. *If condition* (1.10) *holds, then the players' expected payoffs are independent of* b. *Otherwise, a marginal decrease in* b *increases the low-type seller's payoff and decreases buyers' expected payoffs.*

Proof. See Appendix A. □

When search costs decrease, the low-type seller enjoys a direct benefit and increases her search intensity. This increase, however, exacerbates the solicitation curse and, therefore, dampens buyers' incentives to offer a high price. When search costs are relatively large (i.e., condition (1.10) fails), the direct effect dominates and the low-type seller's expected payoff (reservation price) increases. As buyers must make more generous offers, their expected payoffs decrease. If search costs are sufficiently small (i.e., condition (1.10) holds), the indirect negative effect exactly offsets the direct positive effect. Both the low-type seller's expected payoff and buyers' expected payoffs remain unchanged. In the next section, we show that in the non-stationary version of our model, this indirect effect could dominate the direct effect, and thus a reduction in search costs could even strictly decrease the low-type seller's expected payoff.

1.4 Non-Stationary Dynamics

In this section, we consider a non-stationary version of the model and explore another dimension of costly search: dynamics of endogenous search intensity and its impact on equilibrium trading dynamics. The model also generates particular interactions between time-on-the-market and other economic variables and, therefore, contributes to the literature on duration dependence, which we discuss in detail in Section 1.5.

1.4.1 Setup

In this section, we assume that buyers observe how long the seller has stayed on the market (time-on-the-market). In other words, each buyer knows how much time has passed since the seller's arrival. This specification fits well into our continuous-time framework and permits a tractable analysis of non-stationary dynamics. We normalize the seller's arrival time to 0. As specified in Section 1.2, we focus on the binary case where the seller can choose between $\underline{\lambda}$, at no cost, and $\overline{\lambda}$, at flow cost ϕ at each point in time. To minimize triviality as well as make the analysis analogous to the general convex cost case, we restrict attention to the case where ϕ is sufficiently small and $\overline{\lambda}$ is sufficiently large.[13]

For each $a = H, L$, we denote by $p_a(t)$ the type-a seller's reservation price and by $\lambda_a(t)$ her *expected* search intensity. As in the stationary case, in equilibrium no buyer offers strictly above c_H, and thus it is necessarily the case that $p_L(t) < p_H(t) = c_H$ for any t. We apply this result throughout this section. For each $a = H, L$, we let $\sigma_a(t)$ denote the probability that the buyer at time t offers $p_a(t)$. Finally, we represent buyers' unconditional beliefs at time t by $q^u(t)$ and their conditional beliefs by $q^c(t)$.

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[13]The precise necessary and sufficient condition for the subsequent analysis is

$$\phi < \min\left\{\frac{r(\overline{\lambda}-\underline{\lambda})}{r+\underline{\lambda}}(c_H - c_L), \overline{\lambda}(c_H - v_L) - r(v_L - c_L), \frac{r(\overline{\lambda}-\underline{\lambda})}{\underline{\lambda}}(v_L - c_L)\right\},$$

so that if the low-type seller expects to receive c_H from the next buyer, she strictly prefers $\overline{\lambda}$ to $\underline{\lambda}$ and $\overline{p}$ exceeds v_L.

A collection of functions $(p_L(\cdot), \lambda_L(\cdot), \lambda_H(\cdot), \sigma_L(\cdot), \sigma_H(\cdot), q^u(\cdot), q^c(\cdot))$ is a (weak perfect Bayesian) equilibrium if for each $a = H, L$, (i) given $\sigma_H(\cdot)$, $p_a(t)$ is the type-a seller's reservation price and $\lambda_a(t)$ is her optimal search intensity at each time t, (ii) given $p_L(\cdot)$, $p_H(\cdot)$, and $q^c(\cdot)$, $\sigma_a(t) > 0$ only when offering the type-a seller's reservation price is optimal for the buyer at time t, and (iii) given $\sigma_L(\cdot)$, $\sigma_H(\cdot)$, $\lambda_L(\cdot)$, and $\lambda_H(\cdot)$, $q^u(t)$ and $q^c(t)$ are obtained through Bayes' rule.

1.4.2 Exogenous Search Intensity

As a benchmark, we analyze the case where the seller's search intensity is exogenously given. Specifically, we assume that for each $a = H, L$, the type-a seller's search intensity stays constant at λ_a (i.e., $\lambda_a(t) = \lambda_a$ for all t) but allow for type-dependent search intensities (i.e., $\lambda_H \neq \lambda_L$). In order to facilitate comparison to the main model, we focus on the case where λ_L is large enough that the low-type seller strictly prefers accepting c_H from the next buyer to accepting v_L immediately (i.e., $r(v_L - c_L) < \lambda_L(c_H - v_L)$).

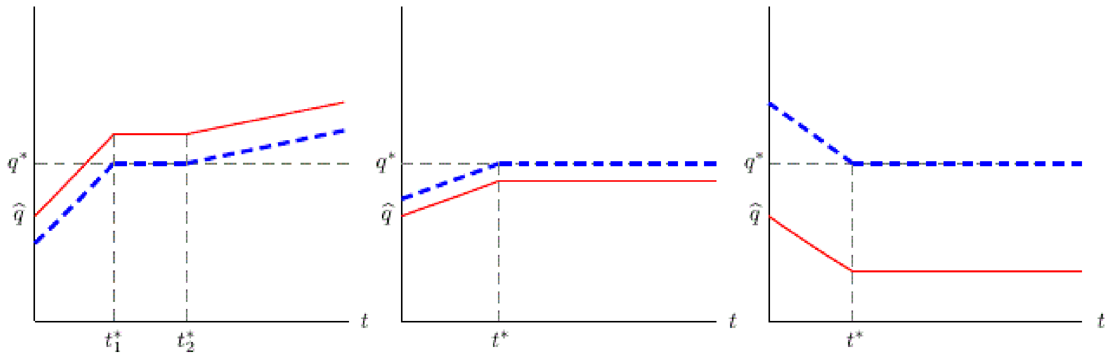
An immediate consequence is that the relationship between buyers' unconditional and conditional beliefs is time-invariant: for all t,

$$q^c(t) = \frac{q^u(t)\lambda_H}{q^u(t)\lambda_H + (1 - q^u(t))\lambda_L} \Leftrightarrow \frac{q^c(t)}{1 - q^c(t)} = \frac{q^u(t)}{1 - q^u(t)} \frac{\lambda_H}{\lambda_L}. \tag{1.11}$$

Clearly, if $\lambda_L > \lambda_H$, then buyers' conditional beliefs $q^c(t)$ are always lower than their unconditional beliefs $q^u(t)$ and, therefore, a seller's arrival (solicitation) is bad news. If $\lambda_L < \lambda_H$, then the opposite is true.

Given this observation, equilibrium exhibits familiar non-stationary properties. Among other things, buyers' (both unconditional and conditional) beliefs are monotone over time

Figure 1.2: Beliefs with Exogenous Search Intensity



Evolution of buyers' unconditional (solid) and conditional (dashed) beliefs in the non-stationary model with exogenous intensity. The left panel is when $\lambda_H < \lambda_L$, the middle panel is when λ_H/λ_L is slightly above 1, and the right panel is when λ_H/λ_L is rather large. q^* is the value such that if $q^c(t) = q^*$, then the buyer's expected payoff by offering c_H is equal to 0 (i.e., $q^* = (c_H - v_L)/(v_H - v_L)$).

and, therefore, their offer strategies take a simple form. The following proposition fully describes the equilibrium outcome.

Proposition 1.3. *In the non-stationary model with exogenous search intensity, the unique equilibrium outcome is given as follows: if $\lambda_L > \lambda_H$, then there exist t_1^* and $t_2^*(\geq t_1^*)$, such that*

- *if $t < t_1^*$, then buyers offer only $p_L(t)(< v_L)$,*
- *if $t \in [t_1^*, t_2^*)$, then buyers offer a losing price (below $p_L(t)$), and*
- *if $t \geq t_2^*$, then buyers offer only c_H.*

If $\lambda_L < \lambda_H$, then there exists t^* such that

- *if $t < t^*$, then buyers either offer only c_H (if λ_H/λ_L is sufficiently large) or offer only $p_L(t)(< v_L)$ (otherwise), and*

- *if $t \geq t^*$, then buyers randomize either between c_H and $p_L(t)(< v_L)$ or among c_H, $v_L(= p_L(t))$, and a losing price.*

Proof. See Appendix A. □

The left panel of Figure 1.2 illustrates equilibrium dynamics when $\lambda_H < \lambda_L$. In this case, the low-type seller not only has a lower reservation price, but also meets buyers at a higher rate than the high-type seller. This implies that the low-type seller necessarily trades faster, and thus buyers' unconditional beliefs $q^u(t)$ always increase over time. Given this observation, it is clear that only the low-type seller trades initially, because $q^c(0) < q^u(0) = \widehat{q} < (c_H - v_L)/(v_H - v_L)$. In addition, if a buyer is willing to offer c_H (at t_2^*), then all subsequent buyers offer only c_H, because they assign even higher probabilities to the high type. Once the game reaches this point, buyers' (both unconditional and conditional) beliefs keep increasing and eventually converge to 1. Now notice that these two phases cannot be adjacent: if they were, the low-type seller would not be willing to accept $p_L(t)$ close to t_1^*. This necessitates losing offers only over the interval $[t_1^*, t_2^*)$. Indeed, the length of the interval $t_2^* - t_1^*$ is determined so that the low-type seller is indifferent between accepting $p_L(t_1^*) = v_L$ at t_1^* and waiting until t_2^* to receive c_H.

The other two panels in Figure 1.2 illustrate the case when $\lambda_H > \lambda_L$.[14] In this case, the two relevant forces for the evolution of buyers' unconditional beliefs work in opposite directions. The fact that $p_L(t) < p_H(t)$ drives $q^u(t)$ up, whereas the fact that $\lambda_L < \lambda_H$

[14]We omit a figure for the case where λ_H/λ_L is sufficiently large, as it is almost identical to the right panel (when λ_H/λ_L is rather large). The only difference is that buyers' conditional beliefs converge to a point below q^*. See the proof of Proposition 1.3 for the exact conditions and full equilibrium characterization.

pushes $q^u(t)$ down. When buyers' conditional beliefs $q^c(t)$ become equal to q^* (at time t^*), these two forces become balanced through buyers' randomization between c_H and $p_L(t) = v_L$: the low-type seller accepts both c_H and v_L but at a lower rate λ_L, whereas the high-type seller accepts only c_H but at a higher rate λ_H. Before time t^*, buyers' conditional beliefs converge to q^*. If λ_H is slightly above λ_L, then buyers' conditional beliefs at time 0 are still below q^* (the middle panel of Figure 1.2). In this case, buyers offer only $p_L(t)$ and, therefore, their beliefs increase over time. If λ_H is sufficiently larger than λ_L, then buyers' conditional beliefs at time 0 are above q^* (the right panel of Figure 1.2). Buyers offer only c_H and, as $\lambda_H > \lambda_L$, buyers' beliefs strictly decrease over time.

1.4.3 Endogenous Search Intensity

We now consider our main model and endogenize the seller's search intensity. We begin with an immediate but crucial observation: the high-type seller always chooses $\lambda_H(t) = \underline{\lambda}$. As in the stationary model, this is because no buyer offers strictly above c_H, and thus the high-type seller has no incentive to increase her search intensity.

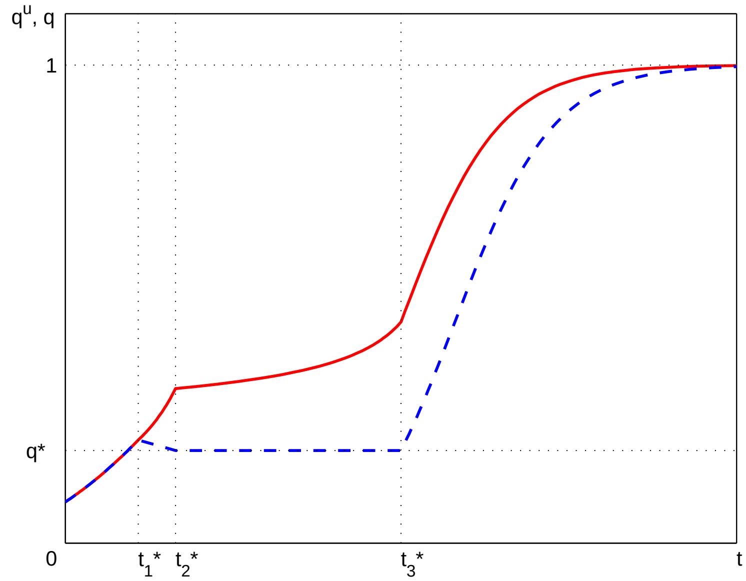
Because $\lambda_L(t) \geq \lambda_H(t) = \underline{\lambda}$ for any t, the basic equilibrium structure is similar to the exogenous-intensity one with $\lambda_L > \lambda_H$. The low-type seller has both a lower reservation price and a weakly higher search intensity than the high-type seller. Therefore, buyers' unconditional beliefs $q^u(t)$ always exceed their conditional beliefs $q^c(t)$ and increase over time. In addition, the following properties hold: early buyers offer only a low price $p_L(t)$ (as $q^c(0) \leq \widehat{q}$), and buyers' beliefs eventually converge to 1. Once buyers offer c_H, their unconditional beliefs strictly increase, which strengthens their incentive to offer c_H.

A non-trivial problem is how the transition from the initial phase (where buyers offer only $p_L(t)$) to the last phase (where buyers offer only c_H) occurs. Endogenous search intensity requires a more subtle transition than the exogenous-intensity case. To see this more concretely, consider the same structure as before, namely that there is an interval $[t_1^*, t_2^*)$ on which buyers offer only a losing price. In this case, the low-type seller has no incentive to increase her search intensity and, therefore, would choose $\underline{\lambda}$ until t_2^*. After t_2^*, she would choose $\overline{\lambda}$, because all buyers offer c_H and, therefore, she has a strong incentive to increase her search intensity. This, however, implies that buyers' conditional beliefs $q^c(t)$ would jump down from q^* to strictly below q^* at t_2^*, in which case buyers would not offer c_H after t_2^* and, therefore, the equilibrium unravels. More generally, buyers' offering c_H provides an incentive for the low-type seller to increase her search intensity, which lowers buyers' conditional beliefs relative to their unconditional beliefs and, therefore, dampens their incentives to offer c_H. This means that it is necessary to find a way to jointly balance buyers' and the low-type seller's incentives: if buyers offer c_H too frequently (infrequently), then the low-type seller chooses too high (low) a search intensity, which eliminates (generates) buyers' incentive to offer c_H.

The following proposition illustrates exactly how the transition occurs in the model with endogenous search intensity. The binary case considered in this section admits a closed-form characterization of the unique equilibrium. The construction, however, is fairly technical. We focus on illustrating the resulting equilibrium dynamics and their implications, relegating an explicit construction to Appendix B.

Proposition 1.4. *In the non-stationary model, there exists a unique equilibrium in which*

Figure 1.3: Beliefs with Endogenous Search Intensity.



Evolution of buyers' unconditional beliefs (solid) and conditional beliefs (dashed). The value q^* represents the point at which a buyer is indifferent between offering c_H and a losing price, that is, $q^* \equiv (c_H - v_L)/(v_H - v_L)$.

there exist three time points, t_1^*, t_2^*, and t_3^*, such that

- *if $t < t_1^*$, then buyers offer only $p_L(t)$ and the low-type seller chooses $\underline{\lambda}$,*
- *if $t \in [t_1^*, t_2^*)$, then buyers randomize between c_H and $p_L(t)$, and the low-type seller chooses $\lambda_L(t) \in [\underline{\lambda}, \overline{\lambda})$,*
- *if $t \in [t_2^*, t_3^*)$, then buyers randomize between c_H and a losing price, and the low-type seller chooses $\lambda_L(t) \in (\underline{\lambda}, \overline{\lambda})$, and*
- *if $t \geq t_3^*$, then buyers offer only c_H, and the low-type seller chooses $\overline{\lambda}$.*

Proof. See Appendix B. □

Figure 1.3 illustrates how buyers' equilibrium beliefs evolve over time. The evolution is more complicated than the exogenous-intensity case, mainly for two reasons. First, although the high type's search intensity stays constant at $\underline{\lambda}$, the low type's search intensity is closely tied to buyers' offer strategies and varies over time. The frequency with which buyers offer c_H increases, inducing the low type to exert more search effort as she stays on the market longer. Precisely, the low type chooses $\overline{\lambda}$ with probability 0 if $t < t_1^*$, with positive and increasing probability if $t \in [t_1^*, t_3^*)$, and with probability 1 if $t \geq t_3^*$. This implies that the solicitation effect intensifies over time. Formally, the relationship between buyers' unconditional beliefs and conditional beliefs is given by

$$\frac{q^c(t)}{1-q^c(t)} = \frac{q^u(t)}{1-q^u(t)} \frac{\underline{\lambda}}{\lambda_L(t)}. \tag{1.12}$$

As $\lambda_L(t)$ increases in t, the relative odds ratio $q^c(t)(1-q^u(t))/(((1-q^c(t))q^u(t))$ decreases in t.

Second, the evolution of buyers' unconditional beliefs depends on the precise details of the players' strategies. To be formal, observe that buyers' unconditional beliefs, in general, evolve according to

$$q^u(t+dt) = \frac{q^u(t)e^{-\underline{\lambda}\sigma_H(t)dt}}{q^u(t)e^{-\underline{\lambda}\sigma_H(t)dt} + (1-q^u(t))e^{-\lambda_L(t)(\sigma_H(t)+\sigma_L(t))dt}},$$

which is equivalent to

$$\dot{q}^u(t+dt) = q^u(t)(1-q^u(t))(\lambda_L(t)(\sigma_H(t)+\sigma_L(t)) - \underline{\lambda}\sigma_H(t)).$$

In the exogenous-intensity case with $\lambda_H < \lambda_L$, buyers' unconditional beliefs rely only on whether trade occurs at c_H or at $p_L(t)$, because $\lambda_L(t) = \lambda_L$ and $\sigma_H(t), \sigma_L(t) \in \{0, 1\}$

with $\sigma_H(t)\sigma_L(t) = 0$ (i.e., buyers offer only $p_L(t)$, a losing price, or c_H) at any t. With endogenous search intensity, $\lambda_L(t)$ is not constant, and there is an interval on which buyers offer multiple prices (c_H and $p_L(t)$ over $[t_1^*, t_2^*)$, and c_H and a losing price over $[t_2^*, t_3^*)$). This complicates the evolution of $q^u(t)$.

In the non-stationary model, the acceleration blessing is best reflected in the fact that buyers' unconditional beliefs $q^u(t)$ always increase and eventually converge to 1. If the low-type seller could not influence her search intensity (i.e., $\lambda_L(t) = \underline{\lambda}$ for any t), then both seller types would eventually trade at the same rate and, therefore, buyers' beliefs would stay constant at an interior level. Endogenous search intensity induces the low type, but not the high type, to trade faster by increasing her search intensity, thereby relaxing future buyers' incentive constraints to offer c_H. As shown above, the same belief convergence occurs in the exogenous-intensity case with $\lambda_L > \lambda_H$. The difference is that the result is driven not by an exogenous assumption, but by the difference in the two seller types' incentives in the current model with endogenous search intensity.

The most notable difference from the model with exogenous search intensity is that buyers' conditional beliefs $q^c(t)$ are not monotone in time. Although $q^c(t)$ still eventually converges to 1, it strictly decreases on the interval $[t_1^*, t_2^*)$ and stays constant on the interval $[t_2^*, t_3^*)$. This is a clear manifestation of the solicitation curse. Although the seller becomes more likely to be the high type over time, the low-type seller also increases her search intensity, which exacerbates the solicitation curse. Over the interval $[t_1^*, t_3^*)$, the solicitation curse more than offsets the acceleration blessing, and thus buyers' conditional beliefs $q^c(t)$ weakly decrease, even though their unconditional beliefs $q^u(t)$ constantly increase.

To better understand why the solicitation curse outweighs the accelerating blessing, recall that buyers are indifferent between c_H and $p_L(t)(< v_L)$ on the interval $[t_1^*, t_2^*)$, and thus

$$q^c(t)(v_H - c_H) + (1 - q^c(t))(v_L - c_H) = (1 - q^c(t))(v_L - p_L(t)).$$

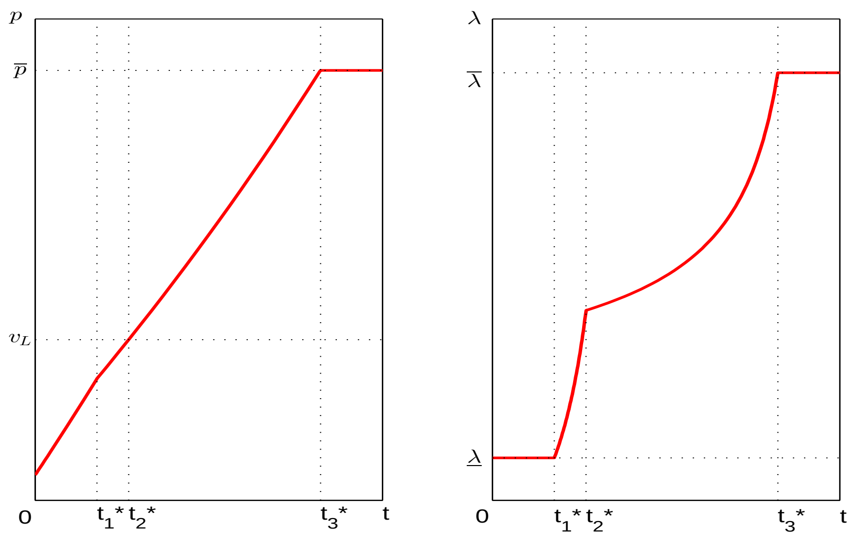
Combining this with equation (1.12) leads to

$$\frac{q^u(t)}{1 - q^u(t)} = \frac{c_H - p_L(t)}{v_H - c_H} \frac{\lambda_L(t)}{\underline{\lambda}}.$$

The left-hand side is necessarily increasing, partly because of the acceleration blessing, whereas the first fraction on the right-hand side is strictly decreasing, because $p_L(t)$ is strictly increasing. In order to preserve the equation, the low-type seller's search intensity $\lambda_L(t)$ must increase sufficiently fast and, in particular, faster than the left-hand side. Combining this observation with equation (1.12), it follows that buyers' conditional beliefs $q^c(t)$ strictly decrease over the region.

Figure 1.4 depicts the paths of the low-type seller's reservation price $p_L(t)$ and equilibrium search intensity $\lambda_L(t)$. Buyers offer c_H with increasing probabilities as the seller's time-on-the-market increases. This causes both $p_L(t)$ and $\lambda_L(t)$ to increase over time. From time t_3^* on, buyers offer c_H with probability 1, and the low-type seller always chooses $\overline{\lambda}$. Time t_1^* is the first point at which the probability of the high type becomes large enough that buyers begin to offer c_H. It follows that $\lambda_L(t)$ also becomes larger than $\underline{\lambda}$ at t_1^*. A crucial time point is t_2^*, which divides the second phase from the third phase. At that point, $p_L(t)$ starts to exceed v_L. This means that a buyer strictly prefers offering $p_L(t)$ to a losing price if $t < t_2^*$, but the opposite is true if $t > t_2^*$. This, in turn, implies that the low-type seller's exit rate switches from $\lambda_L(t)$ (i.e., she trades whenever she meets a buyer)

Figure 1.4: Reservation Price and Search Intensity



Evolution of the low-type seller's reservation price (left) and search intensity (right). $\overline{p}$ is the low-type seller's reservation price if she is sure to receive c_H from the next buyer.

to $\lambda_L(t)\sigma_H(t)$ (i.e., she trades only when she is offered c_H). This explains why buyers' unconditional beliefs increase faster in the second phase than in the third phase in Figure 1.3.

1.4.4 Effects of Reducing Search Costs

We now examine the effects of reducing search costs on the players' payoffs in the non-stationary model. Our main result states that allowing the seller to increase her search intensity can be detrimental to the low-type seller but beneficial to buyers.

Proposition 1.5. *Suppose $r(v_L - c_L) < \underline{\lambda}(c_H - v_L)$. Then, the low-type seller's expected payoff is higher, whereas buyers' expected payoffs, conditional on each t, are lower, when* *the seller is restricted to choose only $\underline{\lambda}$ (i.e., $\overline{\lambda}$ is not allowed).*

Proof. See Appendix A. □

The result for the low-type seller is mainly due to the solicitation curse. When $\overline{\lambda}$ is not available, buyers do not discount their beliefs based on whether they actually face a seller or not. That is, buyers' unconditional beliefs and conditional beliefs coincide. In this case, buyers offer c_H once their unconditional beliefs reach a certain level. If $\overline{\lambda}$ is available, the solicitation curse emerges. Buyers now become more cautious and offer c_H only when their unconditional beliefs reach a higher level, which implies that the seller should wait longer to receive c_H. The desired result follows once this observation is combined with the fact that $p_L(t) \leq v_L$ at the first time when buyers are willing to offer c_H (t_1^* in Proposition 1.4).

The underlying reason why buyers are better off when the seller can increase her search intensity depends on their arrival time. Early buyers, who offer only $p_L(t)$, benefit from the solicitation curse because it lowers the low-type seller's reservation price $p_L(t)$. Late buyers (who arrive after t_3^* in Proposition 1.4) profit from the acceleration blessing. When $\overline{\lambda}$ is not allowed, the two seller types eventually trade at an identical rate, as trade occurs only at c_H later in the game. Therefore, buyers' beliefs cannot exceed a certain level and, under our maintained assumptions, they obtain zero expected payoff (see Proposition 1.3). If $\overline{\lambda}$ is allowed, buyers' beliefs eventually converge to 1, and thus late buyers receive a strictly positive expected payoff.

The assumption of a relatively high baseline search intensity (i.e., $r(v_L - c_L) < \underline{\lambda}(c_H - v_L)$) is necessary for the strong result in Proposition 1.5. In the absence of the assumption, the result is ambiguous, that is, restricting the seller to $\underline{\lambda}$ may or may not benefit the low-type seller and harm all buyers. In particular, if both $\underline{\lambda}$ and $\overline{\lambda}$ are sufficiently small, then the low-type seller's choosing $\overline{\lambda}$ does not have a significant impact on buyers' incentives, and thus the solicitation curse is weak. In this case, the low-type seller's expected payoff can be higher when $\overline{\lambda}$ is available. This implies that early buyers' expected payoffs would be lower, although late buyers' expected payoffs would be still higher, because of the acceleration blessing.

1.5 Discussion

We conclude by explaining the robustness of our main insights and providing further implications of our analysis.

1.5.1 Buyer Inspection

There are various models that allow for buyer inspection, that is, buyers' getting an informative signal about the quality of the good (e.g., Zhu, 2012; Lauermann and Wolinsky, 2016; Kaya and Kim, 2015). We explain how to accommodate buyer inspection within our framework and how our insights extend into such an environment. For simplicity, we restrict attention to the stationary model studied in Section 1.3.

Suppose each buyer receives a signal that is identically and independently drawn from the interval $[\underline{s}, \overline{s}]$ according to the distribution function G_a, where a denotes the quality of the good. Assume that each G_a admits a continuous and positive density g_a. In addition, assume that the likelihood ratio $g_H(s)/g_L(s)$ is strictly increasing (MLRP), $g_H(\underline{s})/g_L(\underline{s}) = 0$, and $g_H(\overline{s})/g_L(\overline{s})$ is arbitrarily large. All other specifications of the environment are identical to those in Section 1.2.

Naturally, buyers' optimal offer strategies involve a cutoff signal: there exists $s^* \in [\underline{s}, \overline{s}]$ such that each buyer offers c_H if and only if his signal is above s^*. For each signal s below s^*, we let $\sigma_L(s)$ represent the probability that buyers offer p_L.

Given s^* and $\sigma_L(\cdot)$, the high-type seller trades at rate $\underline{\lambda}(1 - G_H(s^*))$, whereas the low type trades at rate $\lambda_L((1 - G_L(s^*)) + \int_{\underline{s}}^{s^*} \sigma_L(s)dG_L(s))$. Then, as in Section 1.3, buyers' unconditional beliefs are given by

$$q^u = \frac{\frac{\widehat{q}}{\underline{\lambda}(1-G_H(s^*))}}{\frac{\widehat{q}}{\underline{\lambda}(1-G_H(s^*))} + \frac{1-\widehat{q}}{\lambda_L((1-G_L(s^*))+\int_{\underline{s}}^{s^*} \sigma_L(s)dG_L(s))}}.$$

Notice that, unlike in the baseline model, q^u is not necessarily larger than $\widehat{q}$. This is because the high type generates good signals and, therefore, receives c_H more frequently than the low type (i.e., $1 - G_H(s^*) > 1 - G_L(s^*)$). This provides a countervailing force to the usual effect that the high type accepts only c_H, whereas the low type accepts both c_H and p_L. However, this does not mean that the acceleration effect is absent in this model. It is still present, because without endogenous search intensity (i.e., if $\lambda_L = \underline{\lambda}$), buyers' beliefs would be

$$\frac{\frac{\widehat{q}}{1-G_H(s^*)}}{\frac{\widehat{q}}{1-G_H(s^*)} + \frac{1-\widehat{q}}{(1-G_L(s^*))+\int_{\underline{s}}^{s^*} \sigma_L(s)dG_L(s)}},$$

which is strictly smaller than q^u.

Given q^u and λ_L, buyers' conditional beliefs are given by

$$q^c = \frac{q^u \underline{\lambda}}{q^u \underline{\lambda} + (1 - q^u)\lambda_L} = \frac{\frac{\widehat{q}}{1-G_H(s^*)}}{\frac{\widehat{q}}{1-G_H(s^*)} + \frac{1-\widehat{q}}{1-G_L(s^*)+\int_{\underline{s}}^{s^*} \sigma_L(s)dG_L(s)}}.$$

As in the baseline model, the difference between q^u and q^c captures the solicitation curse, and the acceleration blessing and the solicitation curse exactly offset each other.

1.5.2 More than Two Types

It is well-known that equilibrium characterization becomes significantly more complicated once there are more than two types of sellers. Nevertheless, it is relatively easy to show how the two effects of endogenous search intensity arise in the model with more than two types. For simplicity, we consider the case of three types. The generalization into more types is notationally more cumbersome, but conceptually identical. Again, for simplicity, we consider only the stationary environment in Section 1.3.

Suppose there are three types of sellers: low quality (L), middle quality (M), and high quality (H). For each $a = L, M, H$, denote by c_a a type-a unit's value to the seller and by v_a its value to buyers, and assume that $c_L < c_M < c_H$ and $v_L < v_M < v_H$. Let $\widehat{q}_a$ denote the probability that the seller is of type a at the beginning of the game. The search technology is given as in Section 1.2.

Let p_a denote the reservation price of the type-a seller. The assumption $c_L < c_M < c_H$ guarantees that $p_L < p_M < p_H$. This, in turn, ensures that p_L is accepted only by the low type, p_M by the low type as well as the middle type, and p_H by all three types. It is also clear that each buyer offers either p_H, p_M, p_L, or a losing price. Denote by σ_a the probability that each buyer offers p_a. Finally, denote by λ_a the type-a seller's optimal search intensity. As a lower type gains more from trade and the high type's reservation value is still equal to c_H, $\lambda_L \geq \lambda_M \geq \lambda_H = \underline{\lambda}$.

Let q_a^u represent buyers' unconditional beliefs that the seller is of type a. Following the same steps as in the two-type case,

$$q_L^u = \frac{\frac{\widehat{q}_L}{\lambda_L(\sigma_H+\sigma_M+\sigma_L)}}{\frac{\widehat{q}_L}{\lambda_L(\sigma_H+\sigma_M+\sigma_L)}+\frac{\widehat{q}_M}{\lambda_M(\sigma_H+\sigma_M)}+\frac{\widehat{q}_H}{\underline{\lambda}\sigma_H}},$$
$$q_M^u = \frac{\frac{\widehat{q}_M}{\lambda_M(\sigma_H+\sigma_M)}}{\frac{\widehat{q}_L}{\lambda_L(\sigma_H+\sigma_M+\sigma_L)}+\frac{\widehat{q}_M}{\lambda_M(\sigma_H+\sigma_M)}+\frac{\widehat{q}_H}{\underline{\lambda}\sigma_H}},$$
$$q_H^u = \frac{\frac{\widehat{q}_H}{\underline{\lambda}\sigma_H}}{\frac{\widehat{q}_L}{\lambda_L(\sigma_H+\sigma_M+\sigma_L)}+\frac{\widehat{q}_M}{\lambda_M(\sigma_H+\sigma_M)}+\frac{\widehat{q}_H}{\underline{\lambda}\sigma_H}}.$$

As $\underline{\lambda} < \lambda_M < \lambda_L$, endogenous search intensity clearly lowers q_L^u and increases q_H^u relative to the exogenous case (i.e., the case when $\underline{\lambda} = \lambda_M = \lambda_L$). q_M^u can increase or decrease, depending on the values of λ_L, λ_M, and $\underline{\lambda}$. Nevertheless, it is always the case that q_M^u/q_L^u strictly increases, whereas q_M^u/q_H^u strictly decreases. This shows that the acceleration blessing clearly operates for the case of more than two types.

Let q_a^c denote buyers' conditional beliefs that the seller is of type a. Then,

$$q_L^c = \frac{q_L^u\lambda_L}{q_L^u\lambda_L+q_M^u\lambda_M+q_H^u\underline{\lambda}} = \frac{\frac{\widehat{q}_L}{\sigma_H+\sigma_M+\sigma_L}}{\frac{\widehat{q}_L}{\sigma_H+\sigma_M+\sigma_L}+\frac{\widehat{q}_M}{\sigma_H+\sigma_M}+\frac{\widehat{q}_H}{\sigma_H}},$$
$$q_M^c = \frac{q_M^u\lambda_M}{q_L^u\lambda_L+q_M^u\lambda_M+q_H^u\underline{\lambda}} = \frac{\frac{\widehat{q}_M}{\sigma_H+\sigma_M}}{\frac{\widehat{q}_L}{\sigma_H+\sigma_M+\sigma_L}+\frac{\widehat{q}_M}{\sigma_H+\sigma_M}+\frac{\widehat{q}_H}{\sigma_H}},$$
$$q_H^c = \frac{q_H^u\underline{\lambda}}{q_L^u\lambda_L+q_M^u\lambda_M+q_H^u\underline{\lambda}} = \frac{\frac{\widehat{q}_H}{\sigma_H}}{\frac{\widehat{q}_L}{\sigma_H+\sigma_M+\sigma_L}+\frac{\widehat{q}_M}{\sigma_H+\sigma_M}+\frac{\widehat{q}_H}{\sigma_H}}.$$

Clearly, $q_L^c > q_L^u$ and $q_H^c < q_H^u$. In addition, $q_M^c/q_L^c < q_M^u/q_L^u$, whereas $q_M^c/q_H^c > q_M^u/q_H^u$. This is how the solicitation curse manifests itself in the case of more than two types.

1.5.3 Implications for Duration Dependence

In several markets, each seller's time-on-the-market is publicly available, and its relationship to other economic variables is of interest. In the real estate market, it is a common practice that a seller relists her property only to reset her days on the market (see, e.g., Tucker, Zhang and Zhu, 2013). This indicates that the information has a real economic impact. Indeed, various stylized facts have been well-documented in the literature (see, e.g., Merlo and Ortalo-Magné, 2004). In the labor market, there is a fairly large literature on duration dependence, studying the effects of unemployment duration on employment probabilities.

An intriguing fact is that empirical evidence for prominent patterns is mixed. For example, Lynch (1989) finds evidence for negative duration dependence: a worker's employment probability decreases as his unemployment duration increases. Heckman and Singer (1984), however, observe the opposite pattern. Naturally, the theoretical literature is also divided, some providing a mechanism for one pattern, others explaining the opposite pattern. For example, Vishwanath (1989) and Lockwood (1991) provide an information-based theory to explain why a worker's employment probability may decrease over time, whereas Lentz and Tranæs (2005) present a liquidity-based theory supporting positive duration dependence.

Our non-stationary model contributes to this literature by providing another rationale for positive duration dependence. The high-type seller's probability of trade is strictly increasing over time, as buyers offer the high price with increasing frequency. The low-type seller's rate of trade also increases overall, starting with $\underline{\lambda}$ before t_1^*, and eventually reaching $\overline{\lambda}$ after t_3^*. Our explanation is similar to that of Lentz and Tranæs (2005) in that the result is due to endogenous search intensity. However, our result is driven by buyers' inferences about the seller's type (they assign increasing probabilities to the high type and, therefore, offer the high price more frequently over time), whereas their result is driven by the fact that a risk-averse agent's wealth decreases as he stays unemployed longer.

This implication, however, must be taken with caution for two reasons. First, the low-type seller's probability of trade is not fully monotone over time: it increases from $\underline{\lambda}$ to $\overline{\lambda}$ eventually, but drops at t_2^* (when the low-type seller starts to trade only at c_H). Second, and more importantly, the unconditional probability of trade, which is likely to be the only observable, may exhibit a complex pattern. There are two opposing forces. On the one hand, a remaining seller is increasingly more likely to be the high type over time. As the high type trades at a lower rate than the low type, this puts downward pressure on the unconditional probability of trade. On the other hand, the low-type seller increases her search intensity over time, which tends to increase the unconditional probability of trade. Which force dominates the other depends on the time. The first effect dominates and, therefore, the unconditional probability of trade decreases before t_1^* and after t_3^*. The second effect dominates and the unconditional probability of trade increases between t_1^* and t_2^* and between t_2^* and t_3^*. Note that this result might help explain why empirical evidence for duration dependence is mixed in the literature.

CHAPTER 2
SHOPPING FOR INFORMATION: CONSUMER LEARNING WITH OPTIMAL PRICING AND PRODUCT DESIGN

2.1 Introduction

Consumers often acquire product information in order to make a purchase decision. This can include everything from reading product reviews on websites like Amazon, to looking at quality reviews such as Consumer Reports, to getting advice from friends, to trying out the product in the store. Often, this search for information is an effort to buy a high-quality product and avoid purchasing a low-quality product. The internet has increased access to information, which makes product research even more noteworthy and important to understand. According to "The 2011 Social Shopping Survey" by PowerReviews and the e-tailing group, 50% of consumers spend at least 75% of their total shopping time performing online product research, as opposed to just 21% of consumers in 2010. In fact, 15% of people spend 90% or more of their shopping time on research. The more important or expensive a good, the stronger the incentive to gather information and avoid making a low-quality purchase. A 2013 survey by GE Capital Retail Bank considers items valued at $500 or more, and finds that not only do 81% of consumers do online research, but that they spend an average of 79 days gathering information before making a purchase. A 2016 survey from Ipsos and Zillow indicates how this may translate into actual time spent by asking how many hours of research consumers do before buying. On average, people spend 26 hours for a home, 11 for a car, 5 for a computer, and 4 for a cell phone or tablet.

I study an optimal pricing problem in the presence of consumer product research. A monopolist has a single good for sale which is of either low or high quality. He can choose price and possibly some element of product design before the consumer acts. The consumer begins with a belief that the product is of high quality, which is influenced by brand reputation or advertising. She can choose to buy the product at any time for the posted price, walk away without purchasing the good, or pay to receive some (additional) signal of quality. She receives information via an arithmetic Brownian motion process, where a high-quality good sends better signals, on average. The consumer can collect as much information as she desires before making a purchase decision.

I fully characterize the consumer's optimal information-acquisition strategy, which is an optimal stopping rule in the belief space. More specifically, the buyer sets upper and lower belief boundaries, both of which are absorbing and time invariant. If good signals buoy her belief to the upper boundary, she will buy the good at the posted price, and if bad signals reduce her belief to the lower boundary, she will discontinue search without purchasing the good. The consumer's optimal strategy is therefore to have an interval of beliefs over which she does additional product research.

I investigate the implications of the consumer's product research strategy for the seller's optimal pricing decision. It is clear that if the seller posts a sufficiently high price, the consumer will never buy his product, and if he posts a sufficiently low price, the consumer will buy the product immediately without search. It is unclear, however, under which circumstances the monopolist would prefer the consumer to gather additional information and under which ones he would prefer immediate sale. This is due to his tradeoff between price and probability of sale. With a low enough price, the monopolist can ensure purchase, but if the price is high enough for the consumer to choose to gather information, a series of bad signals may result in no sale.

I show that if the consumer is sufficiently optimistic about product quality, the seller induces product research by posting a high price, while if the consumer is pessimistic, he is more likely to post a low price and sell the good immediately. When belief about quality is high, the seller is optimistic that the signal will reveal that the good is truly of high quality, raising the buyer's willingness to pay, without running too high of a risk that she will walk away without purchasing. When belief is low, however, the seller expects that if the consumer receives the signal, it will reveal his product to be of low quality. He therefore wishes to sell right away to avoid the risk of the consumer not buying at all. I also show a similar result regarding the cost of search and the quality of information. If search cost is below some threshold, or information quality is above some threshold, then inducing search is always the optimal strategy for the seller.

The above result is in direct contrast to the existing one in the literature. Branco et al. (2012) analyze a model in which consumers search for information about a product with many attributes.[1] They can pay a search cost to discover if each one of the unknown attributes has a positive or negative effect on their values, and use an optimal stopping rule to decide when to purchase. In this case, new information affects value directly rather than being a signal or underlying quality as in my model. Branco et al. find that the seller prefers

[1]This baseline model is extended by Ke et al. (2015) and Branco et al. (2015) to incorporate two products and the choice of information quality, respectively.

to encourage search when the initial value is low and to sell immediately when initial value is high. This drastic difference in outcomes is due to the fact that in my model, the seller has an indication of what the signal will reveal, whereas with private values, new information is equally likely to increase or decrease value. Therefore, in their model, the monopolist prefers to capitalize on high current values by selling right away.

Additionally, I analyze the effects of a decrease in search costs (increase in information quality) on the seller's optimal pricing decision.[2] Intuitively, as search becomes less expensive, the consumer will do more of it, which will increase the information rent she receives from the seller, and decrease the price. This reasoning holds when it is optimal for the seller to post a low price and sell immediately. If the seller prefers search, however, I show that it is possible that a decrease in cost will cause the seller to raise the price. When the monopolist desires the consumer to acquire information, if search becomes easier or more appealing, he may raise the price in order to keep incentives balanced and continue to induce product research.

I also study the implications of the consumer's product research strategy on the seller's optimal choice of product design. Following the literature, I assume that the seller can choose the dispersion of product quality as well as price. This aspect of product design is another tool that the seller can use to encourage or discourage search by making it more or less appealing. I show that if the firm wishes to sell the good immediately, it posts a low price and prefers no dispersion of product quality. In contrast, if the seller prefers consumer

[2] I also consider the effects of an increase in prior belief on equilibrium price, and find that price may be discontinuous in belief. When belief becomes large enough for the seller to prefer product research, there is a discrete jump upwards.

search, he posts a relatively high price and desires an intermediate level of dispersion.

The seller's potential desire to choose an interior level of quality dispersion contrasts a common result in the literature that an extremal level of dispersion is always optimal. Previous research (e.g. Johnson and Myatt (2006) and Bar-Isaac et al. (2012)) finds that the seller's optimal level of dispersion is always either as much or as little as possible. Both Johnson and Myatt (2006) and Bar-Isaac et al. (2012) consider consumers whose willingnesses to pay are drawn from a distribution, and the monopolist can control the dispersion of that distribution as well as price. In my model, a firm wishing to sell immediately still desires extreme dispersion. If the seller desires the consumer to gather additional information, however, the optimal level of dispersion is interior. Wang (2016) adds a different kind of consumer search to a framework with a monopoly choice of dispersion and also finds that the optimal level of dispersion may be interior. How product research drives this result is discussed in more detail in Section 2.6.

The analysis of the consumer's problem draws on the literature concerning experimentation with an underlying state. Making use of the real options frameworks of McDonald and Siegel (1986) and Dixit and Pindyck (1994), the literature largely follows Chernoff (1972), who was the first to study the problem of learning about an unknown but constant drift of a Brownian motion process, using the current belief as a sufficient statistic for the history of accumulated information. Others extend this method, including Bernardo and Chowdhry (2002) and Felli and Harris (1996) who study investment and productivity problems, and Bergemann and Välimäki (2000) and Bolton and Harris (1999) who consider social learning of a group of agents, each receiving different signals.

The most related paper from the real options literature is that of Décamps et al. (2005). They also analyze a framework in which the drift of the signal process can have a high or low value. However, in their model, the signal and value of the product the same. This means that the person exercising the option is concerned not only with the belief about the drift of the process but also with the path of the process, or the value itself. Therefore, belief is no longer a sufficient statistic to summarize knowledge, and there is path dependency. To overcome this, they use filtering and martingale techniques rather than dynamic programming. The current model does not suffer from the same path dependency because the Brownian motion process is only a signal of the constant value to the consumer, rather than the value changing with the signal process.

The consumer's problem is also related to the recent literature concerning games of seller reputation with a public signal (e.g. Daley and Green (2012), Gul and Pesendorfer (2012), Bar-Isaac (2003), Kolb (2015a), Kolb (2015b), and Dilmé (2016)). All of these papers analyze the case in which the seller of a good (or political party in Gul and Pesendorfer (2012)) is informed about the quality of the good. He can then choose either how much effort to exert or whether or not to remain in the market as a signal of quality. Except for Kolb (2015b), they also consider a market of short-lived, competitive buyers. This creates an equilibrium structure with either a reflecting or resetting barrier in the belief space. My model differs in that I assume that the seller has no private information, the consumer is strategic and long-lived, and the seller commits to staying in the game.

My analysis of the consumer's optimal product research strategy is particularly close to that of Gul and Pesendorfer (2012), who study the case in which all agents are symmetrically informed about quality. The major difference is that they assume that there are two players that compete and choose to buy information, whereas in my model, only the consumer can receive signals. This disparity creates different optimal strategies for when to stop receiving the signal and therefore end the game. In addition, the competition between the two players constitutes the entire game; there is no player like the monopolist making decisions before the other players act. They therefore do not examine when and how instruments like price might be used to influence agents' information gathering strategies.

The rest of the paper is organized as follows: Section 2.2 presents the underlying environment and considers the consumer's problem. Section 2.3 examines the pricing strategy of the seller. Section 2.4 analyzes the planner's problem. Section 2.5 studies how the optimal strategy is affected by changes in prior belief, cost, and information quality. Section 2.6 extends the model to also incorporate a product design choice. Section 2.7 concludes by presenting a discussion of extensions and applications.

2.2 Consumer's Problem

2.2.1 Environment

Consider a consumer who is considering purchasing a good of unknown quality and can do product research. The good is of either high quality or low quality, and the consumer believes that the good is of high quality with probability $\widehat{q}$.[3] If the product is of high quality,

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[3] Because there is only one consumer, quality can also be interpreted as (idiosyncratic) match value with the probability of a good match being $\widehat{q}$. The model can also be interpreted as a measure 1 continuum of consumers with mass $\widehat{q}$ of them matching well with the product.

it is worth $V_H > 0$ to the consumer, and if it is of low quality, it is worth $V_L < V_H$ to her. Note that no assumption is made about the sign of V_L. The consumer can buy the good for the posted price of P. I assume that the buyer is risk neutral, so that she attains a utility of $V_a - P$ for $a \in \{H, L\}$ at the time of purchase.

At each point in time, the consumer has three choices: she can buy the product, walk away without purchasing the good, or search for more information about the quality, incurring flow cost $c > 0$. If the consumer chooses to walk away, then she receives nothing.[4] If she decides to do research, however, then she obtains more information about the quality of the good through a type-dependent signal. She will use this signal to update her belief about product quality and to determine when she has acquired a sufficient amount of information to buy the good.[5]

If the consumer does research, then information about the good arrives according to the following exogenous process:

$$dX_t = \alpha dt + \sigma dZ_t,$$

where $Z = (Z_t)_{t \geq 0}$ is a standard one-dimensional Brownian motion, with an expected value of 0 for each t. In addition, the process has a positive variance ($\sigma > 0$), and α is a type-dependent drift. If the good is of high quality, α is equal to $\mu > 0$, and if it is of low quality, α is equal to $-\mu$. Therefore, if the good is of high quality, the cumulative

[4]Note that normalizing the consumer's outside option to 0 means that a $V_L < 0$ is simply a low-value good worth less than the outside option.

[5]The consumer's problem is related to the theory of real options. The consumer is holding a call option to irreversibly invest in, or buy, the product for a fixed price. If and when she decides to buy the good, she is giving up the possibility of waiting for new information that might affect her belief about quality.

signal is expected to increase over time, while if the good is of low quality, it is expected to decrease. This indicates that the cumulative signal X_t is normally distributed with mean αt and variance $\sigma^2 t$.

2.2.2 Consumer's Strategy

The consumer's optimal strategy is based on her belief about product quality. All relevant information for the belief is contained in the cumulative signal, X_t.[6] Denoting $\mathcal{F}_t$ as the history, or filtration generated by past observations, the consumer's posterior belief that product quality is high can be obtained by combining Bayes' rule with the distribution of the signal as follows

$$\begin{aligned} q_t = \Pr(\alpha = \mu|\mathcal{F}_t) &= \frac{\widehat{q}\exp\left(-\frac{(X_t-\mu t)^2}{2\sigma^2 t}\right)}{\widehat{q}\exp\left(-\frac{(X_t-\mu t)^2}{2\sigma^2 t}\right) + (1-\widehat{q})\exp\left(-\frac{(X_t+\mu t)^2}{2\sigma^2 t}\right)} \\ &= \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})\exp\left(\frac{-2\mu X_t}{\sigma^2}\right)}. \end{aligned} \tag{2.1}$$

In order for the consumer to construct a strategy based on her belief, she must understand how the marginal signal affects q_t. To do so, she applies Itô's lemma, the stochastic calculus counterpart to the chain rule, to equation (2.1),and obtains the following filtering equation:

$$dq_t = q_t(1-q_t)\frac{2\mu}{\sigma}dZ_t. \tag{2.2}$$

A few things are worth noting. First, because the expected value of Z_t is zero, belief about product quality is a martingale, which is necessary for consistency. Second, how much the consumer adjusts her belief due to new information depends on the current belief. New information matters the most when the consumer is most unsure about the type (i.e. when

[6] See, for example, Chernoff (1972), Chapter 17.

q_t is close to 1/2). Finally, it is useful to define the quality of news by $\gamma \equiv 4\mu^2/\sigma^2$ (meaning that equation (2.2) can be rewritten as $dq_t = q_t(1-q_t)\sqrt{\gamma}dZ_t$). Intuitively, the consumer learns more from search if either goods of different qualities send very different signals (large μ) or there is little noise (small σ). Therefore, equation (2.2) indicates that belief adjusts more to new information when the quality of news is high.

The consumer uses this belief evolution to determine her continuation value $V(q)$. For the remainder of the analysis, drop time subscripts and assume that neither the consumer nor the firm discount future consumption. If the consumer is acquiring information, then her continuation payoff can be written in the following way:

$$\begin{aligned} V(q) &= -cdt + \mathbb{E}V(q+dq) \\ &= -cdt + V(q) + V'(q)\mathbb{E}[dq] + \frac{1}{2}V''(q)\mathbb{E}[(dq)^2]. \end{aligned}$$

For any belief, her value is the flow cost of paying to see the signal, plus how much the value is expected to change due to additional information from that signal. The second term can be rewritten with a Taylor expansion, and simplified by noting that $\mathbb{E}[dq] = 0$ (because belief is a martingale) and $\mathbb{E}[(dq)^2] = (q(1-q)2\mu/\sigma)^2$. Therefore, the relevant stochastic Bellman, or HJB, equation reduces to

$$0 = -c + \frac{\gamma}{2}q^2(1-q)^2V''.$$

It is straightforward to show that the solution to the HJB equation is given by

$$V(q) = \frac{2c}{\gamma}(1-2q)\ln\left(\frac{1-q}{q}\right) + k_2q + k_1, \tag{2.3}$$

where k_1 and k_2 are constants of integration.

The following proposition is a complete characterization of the consumer's optimal information acquisition strategy.

Proposition 2.1. *There exist beliefs* $\overline{q}$ *and* $\underline{q} \leq \overline{q}$ *such that the consumer gathers more information when current belief* q *is in the interval* $(\underline{q}, \overline{q})$. *She walks away without purchase if her belief is less than or equal to* $\underline{q}$, *and buys the good if her belief is greater than or equal to* $\overline{q}$. *These boundaries are identified by the solution to the system of equations:*

$$\begin{aligned} \overline{q}V_H + (1-\overline{q})V_L - P &= \frac{2c}{\gamma}\left((2\overline{q}-1)(\underline{\ln} - \overline{\ln}) + \frac{(1-2\underline{q})(\overline{q}-\underline{q})}{\underline{q}(1-\underline{q})}\right) \\ \underline{q}V_H + (1-\underline{q})V_L - P &= \frac{2c}{\gamma}\left((2\underline{q}-1)(\underline{\ln} - \overline{\ln}) + \frac{(1-2\overline{q})(\overline{q}-\underline{q})}{\overline{q}(1-\overline{q})}\right) \end{aligned} \tag{2.4}$$

where $\overline{\ln} \equiv \ln\left(\frac{1-\overline{q}}{\overline{q}}\right)$ *and* $\underline{\ln} \equiv \ln\left(\frac{1-\underline{q}}{\underline{q}}\right)$.

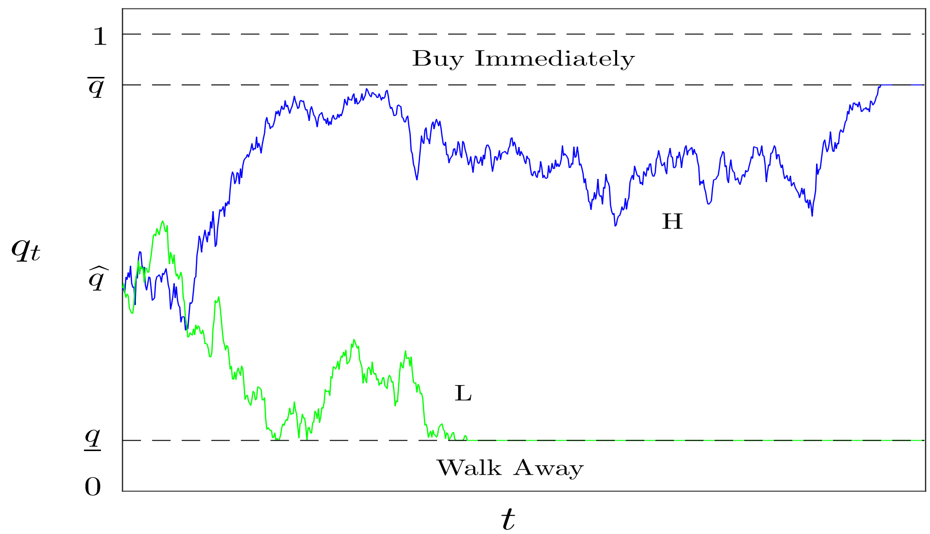
The consumer's optimal strategy is an optimal stopping rule.[7] If her belief ever drops to $\underline{q}$, the consumer walks away, and the game is over. If her belief ever rises to $\overline{q}$, she buys. Two simulations of this strategy are shown in Figure 2.1. The upper line is a possible belief path for a good that is eventually sold, while the lower line represents a possible belief evolution for a product that is not purchased by the consumer.

Four boundary conditions on the value function are required to identify the two constants of integration in equation (2.3) and belief boundaries $\overline{q}$ and $\underline{q}$. The first two are value matching conditions:

$$\begin{aligned} V(\underline{q}) &= 0 \\ V(\overline{q}) &= V_H\overline{q} + V_L(1-\overline{q}) - P. \end{aligned} \tag{2.5}$$

[7]Note that the consumer takes the price as given because the seller commits to it before she acts. Also note that while $\widehat{q}$ does not affect the belief bounds directly (there is path independence), it does affect them indirectly through the equilibrium price, which is endogenized in Section 2.3.

Figure 2.1: Beliefs Over Time



Simulated beliefs when $\alpha = \mu$ (upper) and $\alpha = -\mu$ (lower) with $\gamma = .005$ and $\widehat{q} = .5$.

They say that the consumer's continuation value must be equal to her outside option of 0 when she walks away, and equal to the expected value less the price upon purchase. In other words, $V(q)$ must be continuous. The second two boundary conditions are smooth-pasting conditions:

$$V'(\underline{q}) = 0$$
$$V'(\overline{q}) = V_H - V_L. \quad (2.6)$$

They guarantee the optimality of the consumer's research strategy by ensuring the value function is globally differentiable.[8]

[8]To demonstrate how this guarantees optimality, consider what would happen if $V(q)$ approached $\underline{q}$ with a slope greater than 0, creating a kink. If the consumer chose to continue searching for a short interval of time, Δt, she would observe another signal, and her belief would either be $q' < \underline{q}$ or $q'' > \underline{q}$. The average of these two points would yield a higher continuation value than the kink point itself, indicating that the chosen lower boundary is not the optimal belief at which to stop

Even though explicit solutions for $\underline{q}$ and $\overline{q}$ are not available, it is possible to characterize how they adjust in response to changes in exogenous parameters and price.[9] The following proposition uses the implicit function theorem to summarize potential shifts in $\overline{q}$ and $\underline{q}$.

Proposition 2.2.

(i) An increase in c causes $\overline{q}$ to decrease and $\underline{q}$ to increase, so that $[\underline{q}, \overline{q}]$ contracts,

(ii) an increase in γ causes $\overline{q}$ to increase and $\underline{q}$ to decrease, so that $[\underline{q}, \overline{q}]$ expands,

(iii) an increase in P causes both $\overline{q}$ and $\underline{q}$ to increase, so that $[\underline{q}, \overline{q}]$ shifts up, and

(iv) an increase in V_H or V_L causes both $\overline{q}$ and $\underline{q}$ to decrease, so that $[\underline{q}, \overline{q}]$ shifts down.

Proof. See Appendix. □

The intuition behind the results in Proposition 2.2 is as follows. If the value of information decreases (c increases or γ decreases), then the interval $[\underline{q}, \overline{q}]$ contracts. In others words, as search becomes more expensive, the buyer will do it less; she will buy the good when she is less confident about its quality and walk away when she is more confident about its quality. Additionally, if the net value of the product decreases (P increases or V_H or V_L decreases), then $[\underline{q}, \overline{q}]$ shifts up. The reward for buying a good object is now smaller, so the consumer needs to be more confident in order to purchase the product and

product research. Therefore, it must be that $V'(\underline{q}) = 0$. A similar argument can be applied to the upper boundary $\overline{q}$.

[9] It is common not to obtain explicit solutions for boundaries in problems with learning. For example, Chernoff (1972), Bernardo and Chowdhry (2002), Felli and Harris (1996), Bolton and Harris (1999), and Gul and Pesendorfer (2012) all have models with learning and no explicit solutions for cutoffs.

is less willing to continue learning for a smaller payoff. Understanding how the consumer responds to changes in parameter values will be especially useful in analyzing the seller's decisions in subsequent sections.

2.3 Optimal Pricing

2.3.1 Seller's Problem

The monopolist sets the price $P \geq 0$ at the beginning of the game and has no more information than the consumer about the quality of the good. If value is interpreted as underlying quality, this can be viewed as a situation in which the product is new and untested. If the consumer has an idiosyncratic match value for the good, however, the seller cannot have more information than the consumer. Either way, the marginal cost of production is normalized to 0. In addition, the seller is risk neutral so that his utility is P if the good is purchased and 0 otherwise.

The seller faces the usual tradeoff between price and sale. In typical problems, the quantity the monopolist is able to sell decreases as he increases price. In the current model, the seller's expected profit is the price multiplied by the ex-ante probability of sale. I show below that the probability of sale is decreasing in the price. Therefore, the seller's decision to increase the posted price is based on how much the probability of sale will fall as a result.[10]

Despite the intuitive nature of the tradeoff between price and probability of sale, the seller's optimal pricing decision is complicated by the fact that the probability of sale is

[10] If the single consumer is interpreted as a continuum of consumers with measure 1, then the tradeoff will be between price and the expected number of consumers that will buy the good.

affected by the consumer's optimal strategy. Raising the price will decrease the chance of sale precisely because the consumer adjusts her behavior by choosing to be more confident in the good's quality both when she buys and walks away.

Taking all of this into account, firm profit is defined by the following Lemma.

Lemma 2.1. *The seller's expected profit is*

$$\Pi = \frac{\widehat{q} - \underline{q}(P)}{\overline{q}(P) - \underline{q}(P)} P, \tag{2.7}$$

where $\widehat{q}$ *is the initial belief that the product is of high quality, and the consumer's behavior is a function of price.*

Proof. See Appendix. □

The first term is the likelihood of sale, or the chance that belief increases $\overline{q} - \widehat{q}$ before it decreases $\widehat{q} - \underline{q}$. In other words, it is the chance that the upper bound, $\overline{q}$, is hit before the lower bound, $\underline{q}$. Therefore, increasing price (shifting $[\underline{q}, \overline{q}]$ up) lowers the probability of sale by moving $\widehat{q}$ relatively closer to the lower bound.

If the seller posts an interior price, it will be characterized by the first-order condition. This price (P^*) will dictate a consumer strategy ($\underline{q}^*, \overline{q}^*$). The equilibrium price and belief bounds are the solution to the three-equation system of equation (2.4) and the following first-order condition of profit:[11]

$$0 = \frac{\widehat{q} - \underline{q}^*}{\overline{q}^* - \underline{q}^*} - P^* \frac{\gamma}{2c} \frac{(\overline{q}^* - \widehat{q})(\underline{q}^*)^2(1 - \underline{q}^*)^2 + (\widehat{q} - \underline{q}^*)(\overline{q}^*)^2(1 - \overline{q}^*)^2}{(\overline{q}^* - \underline{q}^*)^3}. \tag{2.8}$$

[11]Note that the existence or uniqueness of a solution to this system is not guaranteed because as discussed later in this section, the profit function may not be single-peaked. Throughout the paper, P^* refers to the interior solution, if it exists, that yields the most profit.

2.3.2 Boundary Pricing

The range of prices that might maximize the seller's profit is affected by the consumer's behavior. If the price is very high, the consumer will walk away immediately, and if the price is very low, she may buy the good immediately. She will only search for additional information when price is neither too high nor too low and $\widehat{q} \in [\underline{q}, \overline{q}]$.

Define P^W as the price at which the consumer is indifferent between walking away and searching and P^B as the price at which she is indifferent between doing product research and buying immediately. At a price of P^B, it must be that $\overline{q} = \widehat{q}$.[12] Plugging this information into system (2.4) yields

$$P^B = \widehat{q}V_H + (1-\widehat{q})V_L - \frac{2c}{\gamma}\left(-(1-2\widehat{q})(\underline{\ln}^B - \widehat{\ln}) + \frac{(\widehat{q}-\underline{q}^B)(1-2\underline{q}^B)}{\underline{q}^B(1-\underline{q}^B)}\right), \quad (2.9)$$

where $\underline{q}^B$ is the lower bound associated with $\overline{q} = \widehat{q}$, $\underline{\ln}^B = \ln\left(\frac{1-\underline{q}^B}{\underline{q}^B}\right)$, and $\underline{q}^B$ is uniquely defined by[13]

$$\frac{2c}{\gamma}\left(\frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)} + 2\underline{\ln}^B\right) = \frac{2c}{\gamma}\left(\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})} + 2\widehat{\ln}\right) + V_H - V_L.$$

Notice that the second term of equation (2.9) is always negative. Then, the price can be interpreted as the expected value the consumer attains upon purchase, less the option value of product research, which she gives up. Therefore, the more valuable search is to the consumer, the more she must be compensated to buy immediately. Note that she receives rent in the form of a lower price not from information that she has at the beginning of the

[12] Similarly, P^W is defined where $\widehat{q} = \underline{q}$ and system (2.4), but the statement of it is omitted because P^W is absent in later analysis.

[13] Note that the right-hand side of the equation is constant, while the left-hand side is strictly decreasing, yielding a unique $\underline{q}^B$, and therefore a unique P^B.

game, but rather from her potential to acquire it.

It is worth noting that one or both of these prices can be negative. If $P^B < 0$, the consumer is unwilling to purchase the good immediately at any feasible price, and if $P^W < 0$, no price is low enough to induce the consumer to ever buy. Overall, the shopper will search for more information if $P \in (\max\{0, P^B\}, P^W)$.[14]

It is clear that it is never optimal for the monopolist to charge P^W, but it may be optimal for him to charge a price of P^B. P^W is ruled out because as $P \to P^W$, the probability of sale and profit approach 0. A price of P^B will be ideal when the firm wishes to avoid consumer research altogether and sell the good immediately. The seller's choice is therefore between posting a low price (P^B) to sell immediately or posting a high price (P^*) to induce the consumer to search.

For subsequent analysis, it is useful to understand the comparative statics of P^B with respect to cost, information quality, and prior belief, as summarized below.

Proposition 2.3. *The price at which the consumer is indifferent between purchasing the good immediately and gathering more information, P^B, is increasing in prior belief ($\widehat{q}$) and cost (c), while it is decreasing in information quality (γ).*

Proof. See Appendix. □

As noted above, P^B is the initial expected value minus the option value of gathering additional information. A high option value lowers price because the consumer must be

[14]Note that each price is associated with a strategy $\{\underline{q}, \overline{q}\}$ (from system (2.4)). This relationship between price and strategy is not dependent on the prior $\widehat{q}$. Which prices (and therefore which $\{\underline{q}, \overline{q}\}$'s) are relevant for the firm's problem, however, depend on $\widehat{q}$.

compensated more for choosing not to search. Two opposing factors influence how P^B changes when c $(-\gamma)$ increases. On the one hand, if the consumer were to search for additional information, she would search less ($\widehat{q}$ and $\underline{q}^B$ are closer together) and therefore learn less, decreasing the option value and putting upward pressure on price. On the other hand, it is more expensive for the seller to compensate the consumer for not doing product research, which puts downward pressure on the price. Overall, however, the first motive dominates, so that the option value decreases and the price increases. When $\widehat{q}$ increases, the value of buying immediately also increases, but the option value could increase or decrease, depending on how much the consumer's strategy changes. In the end, however, the rise in initial expected value outweighs any effect the option value has, and price increases.

2.3.3 Pricing for Product Research

It is import to understand under what circumstances the seller finds it optimal to incentivize consumer research by posting a relatively high price of P^*.

In general, the seller prefers to post a low price and sell immediately if prior belief is low and prefers to post a high price, encouraging search, if prior belief is high. Intuitively, if prior belief about product quality is low, the good is likely to be of low quality and send negative signals. It is therefore risky for the seller to allow product research because it is relatively more likely that negative signals depress belief, causing the consumer to walk away without purchase. The seller avoids this risk by posting a low price and selling the product immediately. If prior belief about quality is high, however, then the seller expects the signals to be positive, and search is relatively less risky. Even though it is still possible that negative signals cause the consumer to walk away without purchase, it is less likely. In this case, the seller feels confident enough to post a high price and induce search.

To understand when the seller prefers information acquisition, consider the following sufficient condition for product research,[15]

$$1 - P^B \frac{\widehat{q}^2(1-\widehat{q})^2}{(\widehat{q}-\underline{q}^B)^2} > 0, \tag{2.10}$$

that profit is increasing at $\max\{0, P^B\}$.[16] Equation (2.10) exemplifies the monopolist's tradeoff between the price and the probability of sale. Charging a higher price benefits the monopolist, but also causes the probability of purchase to fall. The second term on the left-hand side of equation (2.10) is the price multiplied by the derivative of the probability of sale with respect to price at P^B. Therefore, for the monopolist to desire product research, either P^B must be low enough, or the probability of sale must fall slowly enough when price is raised from P^B.

Proposition 2.4 characterizes when the sufficient condition holds as prior belief changes.

Proposition 2.4. *As $\widehat{q} \to 1$, the sufficient condition holds, and the seller prefers the consumer to acquire information. As $\widehat{q} \to 0$, the sufficient condition holds $\Leftrightarrow V_L < 0$.*

Proof. See Appendix. □

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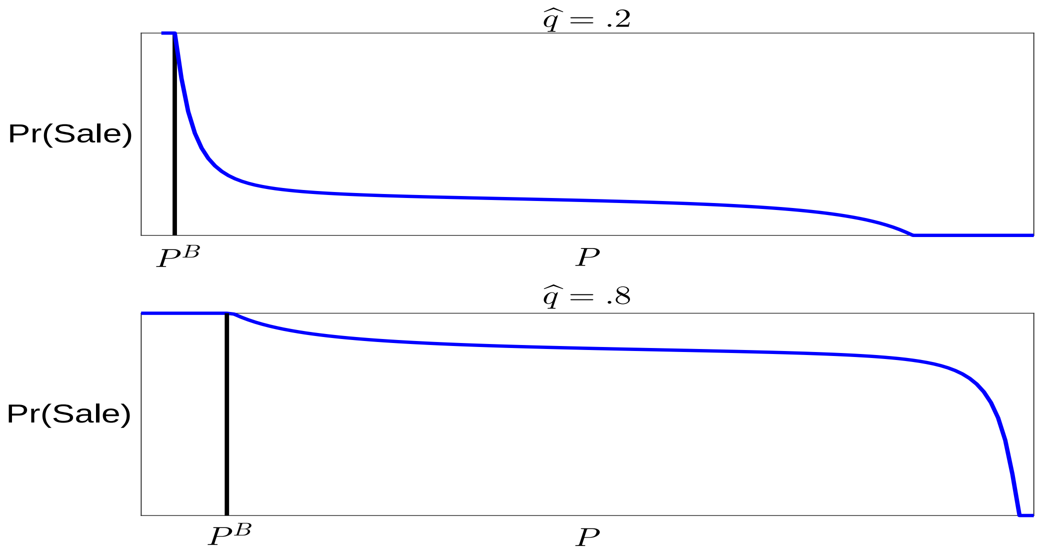
[15]Fully characterizing when the seller prefers search is complicated by the fact that the profit function may not be concave, as seen in Figure 2.3.

[16]Note that profit is always increasing at 0 so the sufficient condition reduces to $P^B \leq 0$ or $P^B > 0$ and equation (2.10).

Figure 2.2: Probability of Sale



$c = 2$, $\gamma = 16$, $V_H = 16$, and $V_L = 2$.

To understand Proposition 2.4, consider how the seller's two motives are affected by a change in $\widehat{q}$, or reputation. First, P^B increases with $\widehat{q}$, so that the highest buy-immediately price is around $\widehat{q} = 1$. This gives the seller the strongest incentive to induce search when he cannot sell the good for a high price initially, or near $\widehat{q} = 0$. Second, how much the probability of sale drops due to product research may change nonmonotonically in prior belief. It is certain, however, that it drops a negligible amount as $\widehat{q} \to 1$, but drastically as $\widehat{q} \to 0$. This gives the seller the strongest incentive to induce search near $\widehat{q} = 1$ and the least incentive near $\widehat{q} = 0$, where product research is the riskiest. Therefore, at least at the extreme values, the seller's two motives work against one another.

To solidify intuition, consider Figure 2.2 depicting the probability of sale for different values of $\widehat{q}$. It is clear that the drop in likelihood of sale is much steeper for a low reputation like $\widehat{q} = .2$ than for a high reputation like $\widehat{q} = .8$ when price is raised form P^B to induce search. This indicates that allowing product research when prior belief is low is much riskier than allowing it when belief is relatively high because the firm is much more likely to lose the sale.

Proposition 2.4 demonstrates that when $\widehat{q}$ is high, the seller's dominant motive is the probability of sale. When raising price results in only a small drop in the probability of sale, inciting product research is "cheap" enough. In other words, even though the seller can guarantee himself a high price by charging P^B, he prefers to induce search. For low values of $\widehat{q}$, the probability of sale motive is again dominant if $V_L > 0$; even though the monopolist cannot guarantee himself a high price, it may still be better to sell immediately because the alternative involves too much risk that the consumer becomes discouraged and walks away without purchasing. If V_L is less than the cost of production, however, it is not possible to sell immediately for very low priors because P^B is negative. In this case, product research is the only way to attain a positive expected profit.

Similar intuition can be used to understand the following proposition, which characterizes when the sufficient condition holds as the cost of search or quality of information changes.

Proposition 2.5. *If $V_L \leq 0$, then $\exists\, c'$ (γ') such that the seller prefers the consumer to gather information $\forall\, c \leq c'$ $(\gamma \geq \gamma')$. If $V_L > 0$ and $\widehat{q} > 1 - V_L^{-1/2}$, then $\exists\, c''$ (γ'') such that the seller prefers the consumer to gather information $\forall\, c \leq c''$ $(\gamma \geq \gamma'')$.*

Proof. See Appendix. □

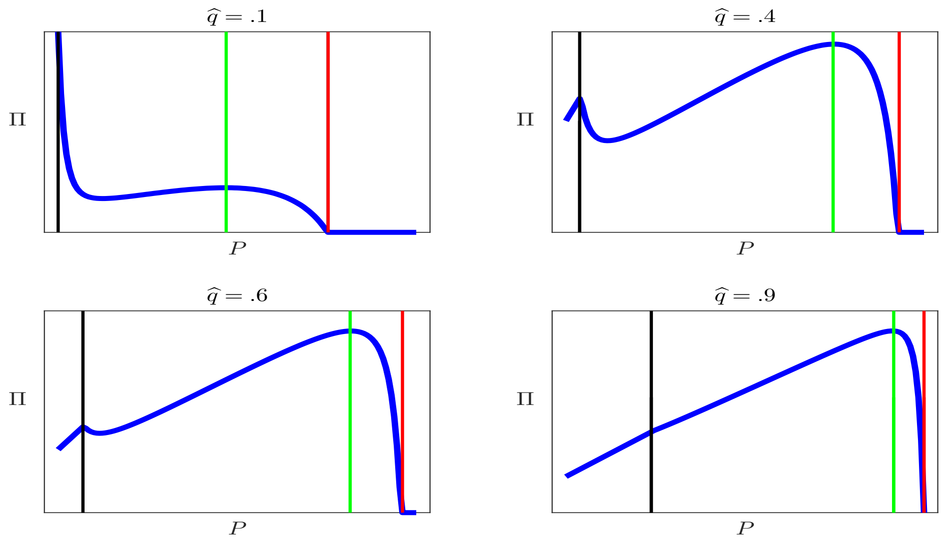
Consider the seller's two incentives as cost increases (quality of information decreases). First, as search costs increase, so does P^B, which makes immediate sale more attractive to the seller. Second, the probability of sale drops more quickly for higher costs. As seen in the proof of Proposition 2.3, $d\underline{q}^B/dc > 0$, so that as cost increases, $\widehat{q}$ and $\underline{q}^B$ become closer together. Therefore, if price were to increase marginally from P^B, the upper and lower bounds would be nearer to one another for high costs, depressing the probability of sale. Overall, the seller's motives work in the same direction and make the corner solution more appealing as cost rises. This yields clear cutoffs under which search is preferable.

To better understand the result, consider the extreme cases. As $c \to \infty$, search will never happen in equilibrium. As $c \to 0$, $P^B \to V_L$. This is not a feasible price if $V_L < 0$, making product research preferable to the seller. If $V_L > 0$, however, the sufficient condition holds only if reputation is sufficiently high. The intuition is the same as that of Proposition 2.4; a low enough $\widehat{q}$ could cause the seller to post P^B and sell immediately because inducing product research is risky.

The above analysis can be summarized by the following intuition. If the firm is not very optimistic about the quality of the good, it has an incentive to sell immediately because if the consumer is allowed to search, she is likely to get bad signals and walk away without purchasing. If the firm is more confident that the product is good, however, it has an incentive to allow the consumer to search. If she does search, she will likely get good signals and want to buy the good, allowing the firm to charge a higher price upon purchase.

This intuition remains relevant even when the sufficient condition does not hold, as in the first three panels of Figure 2.3. In this case, the profit function may not be concave or

Figure 2.3: Profit for Different Prior Beliefs



Profit for $\widehat{q}$'s. The black (left) line is P^B, the green (middle) P^*, and the red (right) P^W. $c = 2$, $\gamma = 16$, $V_H = 16$, and $V_L = 2$.

even single-peaked.[17] The relative strengths of the monopolist's two incentives can be seen in the sizes of profit at P^B and at the interior maximum, P^*. For low prior beliefs, it is better for the firm to charge P^B. As $\widehat{q}$ grows, expected profit from the interior maximum grows as well, and eventually, allowing the consumer to search becomes the profit-maximizing strategy for the firm.

As noted earlier, the result that product research is generally more desirable for higher prior beliefs stands in direct contrast to the result in Branco et al. (2012). In their model, the monopolist chooses to sell right away when value is high because he does not know if the consumer's value will decrease or increase if she is allowed to search. In my

[17]Note that the top right and bottom left panels of Figure 2.3 clearly demonstrate why equation (2.10) is sufficient but not necessary for an interior solution.

model with underlying values, however, the firm has an indication of what the signal will reveal. The firm therefore wishes to capitalize on high beliefs by inducing product research and wishes avoid the signal for low beliefs by posting a low price.

2.4 Planner's Problem

Consider a social planner who takes consumer behavior as given and sets the price at time 0. Denote the socially efficient price as P^S. If V_L is larger than the production cost of 0, then $P^S \leq P^B$ so that the good is transferred to the consumer immediately, avoiding socially wasteful search. Therefore, any amount of search is inefficiently large when $V_L > 0$. If $V_L < 0$, however, information acquisition may be efficient because if $P^B < 0$, the good cannot be sold immediately. In order to understand when product research is efficient, consider the social surplus,

$$\begin{aligned} S &= \text{Pr(sale)}\mathbb{E}[\text{value}] - c\mathbb{E}[\tau] \\ &= \frac{\widehat{q} - \underline{q}}{\overline{q} - \underline{q}}(\overline{q}V_H + (1-\overline{q})V_L) \\ &- \frac{2c}{\gamma(\overline{q} - \underline{q})}\left((\overline{q} - \underline{q})(2\widehat{q} - 1)\widehat{\ln} - (\widehat{q} - \underline{q})(2\overline{q} - 1)\overline{\ln} - (\overline{q} - \widehat{q})(2\underline{q} - 1)\underline{\ln}\right), \end{aligned} \tag{2.11}$$

where $\widehat{\ln} = \ln((1-\widehat{q})/\widehat{q})$ and τ is the time at which either the upper or the lower bound is hit, and the game ends. The planner, like the seller, anticipates the consumer's strategic response to any posted price.

The following is the first-order condition of the planner's problem and shows that the socially efficient price is the lowest feasible price.

$$\frac{dS}{dP} = -\frac{P}{(\overline{q} - \underline{q})^2}\left(\frac{d\overline{q}}{dP}(\widehat{q} - \underline{q}) + \frac{d\underline{q}}{dP}(\overline{q} - \widehat{q})\right) \tag{2.12}$$

Equation (2.12) is simply the derivative of (2.11) with information from the consumer's optimal behavior substituted in. Recall that the interval $[\underline{q}, \overline{q}]$ shifts up as price increases, indicating that (2.12) is weakly negative. This is because given consumer behavior, surplus is maximized when the probability of sale is maximized. The chance of a sale is decreasing in price, so that the socially efficient price will be the lowest feasible price, or $\max\{0, P^B\}$. The planner would like to sell the good immediately if $P^B > 0$ to avoid socially wasteful search.[18] Barring that, he prefers the good to be sold at cost (0), and the consumer to acquire additional information.

If $V_L < 0$, search is efficient only if it is cheap enough or the prior belief is high enough, as summarized below:[19]

Proposition 2.6. *If* $V_L < 0$

(i) and $\widehat{q}V_H + (1 - \widehat{q})V_L \leq 0$*, then* $P^S = 0 \; \forall c$*,*

(ii) and $\widehat{q}V_H + (1 - \widehat{q})V_L > 0$*, then there exists* c^* *(*γ^**) such that* $P^S = 0 \; \forall c \leq c^*$ *(*$\gamma \geq \gamma^*$*) and* $P^S = P^B \; \forall \, c > c^*$ *(*$\gamma < \gamma^*$*),*

(iii) then there exists $\widehat{q}^*$ *such that* $P^S = 0 \; \forall \widehat{q} \leq \widehat{q}^*$ *and* $P^S = P^B \; \forall \widehat{q} > \widehat{q}^*$*.*

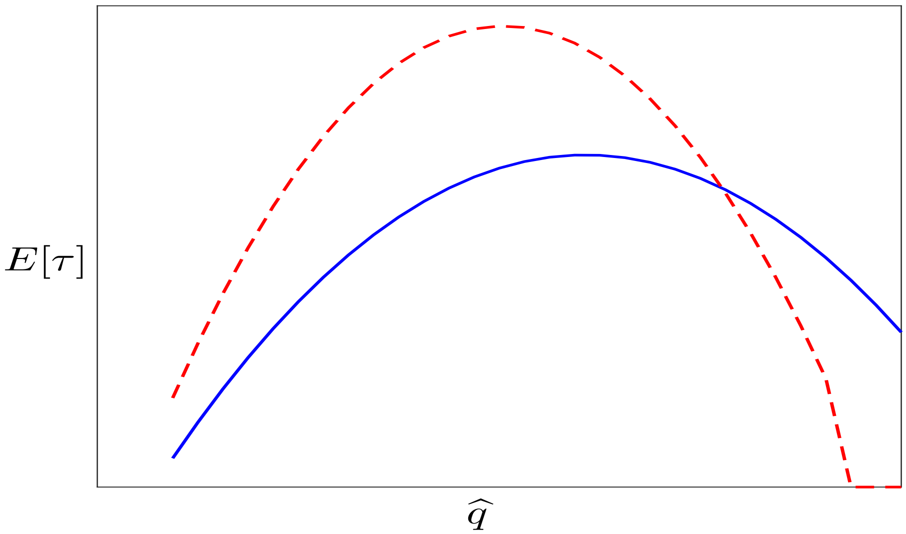
Proposition 2.6 states that the planner prefers the consumer to gather information only when the search cost or the prior belief is too low for the good to be sold immediately.

On the extensive margin, there is an inefficiently large amount of search in equilibrium. To see this, recall that the monopolist also has a cost cutoff under which search is

[18] Note that P^B is bounded below by V_L so that if V_L is positive, then P^B is as well, regardless of other parameter values.

[19] Recall that P^B moves monotonically in all parameters. That guarantees the existence of cutoff values for cost, information quality, and prior belief.

Figure 2.4: Expected Time of Search



Monopolist (solid) vs. Planner (dashed) $\mathbb{E}[\tau]$. $c = 2$, $\gamma = 7.11$, $V_H = 5$, and $V_L = -5$.

optimal, c'. Comparing them, $c^* < c'$, indicating that the monopolist never prefers to induce product research when the planner does not. Therefore, there is an inefficiently large amount of search on the extensive margin. In addition, because charging a price of 0 is never profit-maximizing, the monopolist charges a weakly higher price than the planner.

When both the planner and the monopolist prefer the consumer to acquire information, however, it is difficult to say which solution yields "more" search. A natural measure of the quantity of search is the ex-ante expected time that the consumer will spend gathering information. This, however, may be higher for the monopolist or the planner, depending on the parameter values, as seen in Figure 2.4. In this particular case, if $\widehat{q}$ is small, the planner's solution yields more search in expectation, while if $\widehat{q}$ is relatively high, the opposite is true.

2.5 Changes in Reputation and Cost of Search

2.5.1 Effects of a Higher Prior Belief

Given how crucial the reputation of the firm is to its pricing decision, it is of interest how changes in reputation affect the firm's optimal price and profit. From the above discussion, we already know that P^B increases in $\widehat{q}$.

The following proposition shows that interior price and profit increase in reputation as well.

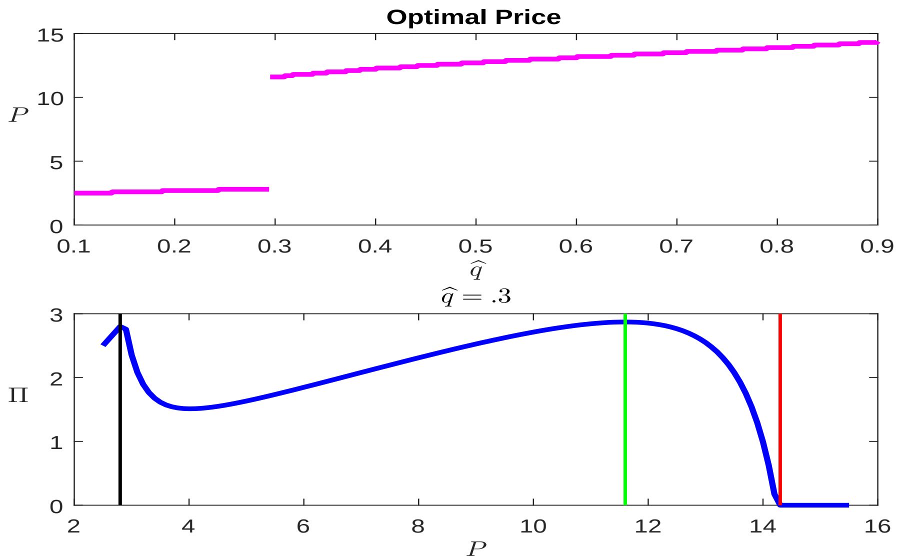
Proposition 2.7. *Both optimal profit,* $\overline{\Pi} \equiv \max\left\{P^B, \Pi^* \equiv P^* \frac{\widehat{q}-\underline{q}^*}{\overline{q}^*-\underline{q}^*}\right\}$*, and interior price,* P^**, are increasing in* $\widehat{q}$*.*

Proof. See Appendix. □

These results are intuitive. A higher $\widehat{q}$ allows the seller to capitalize on the fact that the signal will likely indicate that the product is of high quality by charging a higher price. Therefore, regardless of whether the optimal solution involves search or not, the price charged by the seller will be increasing in belief or reputation.

It need not be the case, however, that price be a continuous function of $\widehat{q}$. For the same parameter values as used in Figure 2.3, the optimal price as a function of reputation is pictured in the top panel of Figure 2.5. For low enough beliefs, the firm prefers to set a low price so that the consumer purchases immediately. For high beliefs, the firm prefers to induce search. The top panel shows that for these parameters, the switch from the corner solution to the interior solution occurs around $\widehat{q} = .3$, while the bottom panel illustrates how at that belief, P^B and P^* yield roughly the same profit.

Figure 2.5: Optimal Price



Profit increases in $\widehat{q}$ because as the consumer becomes more confident about product quality, she is willing to pay more. We have already seen that P^B is increasing in reputation. To see why Π^* is also increasing in $\widehat{q}$, apply the envelope theorem, which yields $d\Pi/d\widehat{q} = P^*/(\overline{q}^* - \underline{q}^*) > 0$. In other words, higher expected values result in higher profit.

2.5.2 Effects of Higher Search Costs (Less Informative Search)

It is natural to consider how changes in search costs affect price and profit, especially since the internet has made information acquisition significantly easier. Above, I showed that P^B increases in $c(-\gamma)$ because as costs rise, the option to search becomes less valuable. Analysis of how P^* reacts to a change in c or γ, however, will be significantly more complicated. Before turning to the specifics, first consider the intuition.

At first glance, one might be tempted to assume that equilibrium price should increase with cost. When costs go down, the consumer is better informed about product quality and therefore requires more information rent from the producer. This pushes price down. When the optimal price is P^B, this is surely the case. When the optimal price is P^*, however, a change in cost or information quality changes the consumer's search behavior, which in turn affects the seller's incentives. It will be easiest to form intuition by disentangling the effects of the movement of $\underline{q}$ and the movement of $\overline{q}$.

Examining how a movement in $\underline{q}$ affects the seller's motives shows that a change in $\underline{q}$ always pushes optimal price in the opposite direction. If the consumer's belief hits the lower bound, she walks away, and the seller receives nothing. Therefore, a movement of the lower bound only affects the seller's incentives through the probability of sale. Think about a marginal increase in the lower bound from $\underline{q}'$ to $\underline{q}''$, so that the solution remains interior ($\widehat{q} \geq \underline{q}''$). This change means that the consumer is more likely to walk away without buying, leaving the seller with nothing. The seller therefore has an incentive to lower the price (and therefore $\underline{q}''$) in order to induce search and increase the probability of sale. The opposite is true if the lower bounds decreases. In this case, the seller takes advantage of the increased probability of sale by raising the price. Either way, a change in $\underline{q}$ puts pressure on the optimal price in the opposite direction ($\underline{q} \uparrow \Rightarrow$ price $\downarrow$).[20]

Next, consider how a change in the upper bound affects pricing decisions. This effect is more complicated because the seller wants belief to hit the upper bound. He now faces a tradeoff between the price and the probability of trade. Think about a marginal decrease in the upper bound from $\overline{q}'$ to $\overline{q}''$, where the solution remains interior ($\widehat{q} \leq \overline{q}''$).

[20]Note that a larger change to $\underline{q}''' \geq \widehat{q}$ also puts downward pressure on price.

The seller has two conflicting motives. On the one hand, he would like to take advantage of the higher probability of sale by raising the price. On the other hand, he has the option to ensure sale by lowering the price (and therefore $\overline{q}$) *even* further. Whether the change in the upper bound will put upward or downward pressure on the price will depend on which of these motives is stronger.[21]

This intuition is formalized for interior solutions in the following proposition.

Proposition 2.8. *The impact on P^* of a marginal change in c will have the same sign as*

$$\frac{1}{c}(\widehat{q}-\underline{q})(\overline{q}-\underline{q})\left((\widehat{q}-\underline{q})\frac{\partial\overline{q}}{\partial P}+(\overline{q}-\widehat{q})\frac{\partial\underline{q}}{\partial P}\right)-\frac{d\underline{q}}{dc}h(\overline{q},\underline{q},\widehat{q},\gamma,c)-\frac{d\overline{q}}{dc}g(\overline{q},\underline{q},\widehat{q},\gamma,c) \tag{2.13}$$

where $h(\cdot)$ and $g(\cdot)$ are functions defined in the Appendix. The impact of a marginal change in γ is similar and relegated to the Appendix.

Proof. See Appendix. □

Equation (2.13) helps decompose the effects of information rent and the upper and lower bounds on the consumer's strategy. The first term characterizes how higher costs reduce information rent by putting upward pressure on the price. To see how the lower bound affects the seller's pricing incentive, first recognize that $h(\cdot)$ is always positive. This means that a change in cost will put pressure in opposite directions on $\underline{q}$ and P^*, as suggested by the intuition above. $g(\cdot)$ is less clear. At low values of $\widehat{q}$, it is certainly negative, but at high values of $\widehat{q}$, it is nearly always positive. This indicates that when the seller's reputation is

[21]Note that a larger change to $\overline{q}''' \leq \widehat{q}$ means the consumer buys immediately so that the seller only has an incentive to raise price.

low, the motive to ensure purchase dominates, while when reputation is high, the motive to take advantage of increased probability of sale by raising price is stronger.

Proposition 2.8 indicates that optimal price may be decreasing in $c(-\gamma)$, especially when the seller's primary motive is to ensure sale. Intuitively, if the monopolist prefers the consumer to acquire information (i.e. the optimal price is P^*), then as search becomes less appealing (more costly or less informative), he must lower the price in order to compensate and induce product research.

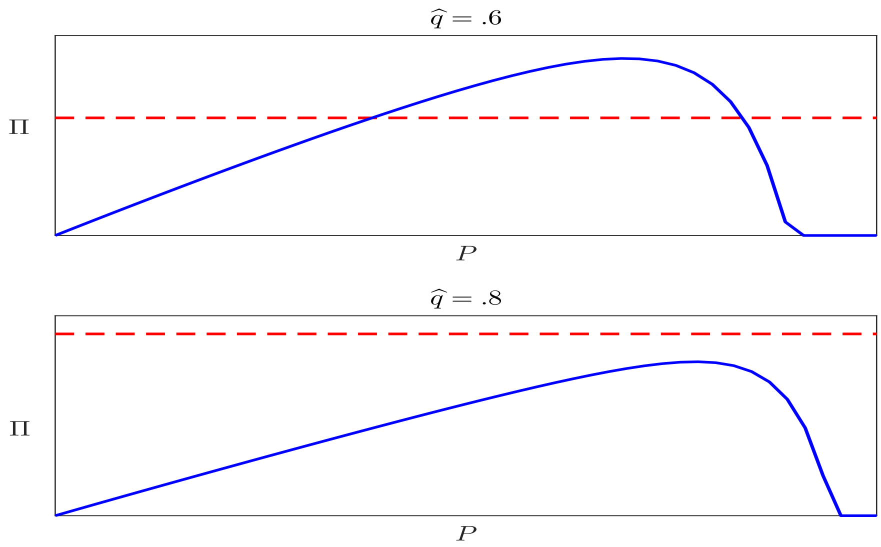
Combining these results indicates that overall, the effect on price of an decrease in search costs is ambiguous. When the good is sold immediately, price decreases with cost, but when the consumer gets additional information, price can increase when cost decreases. The fact that the consumer can choose whether to acquire additional information is the driving force for this result. If the monopolist desires the consumer to search, he must raise the price in order to keep incentives balanced when product research becomes cheaper or more appealing.

Price changing ambiguously with $c(-\gamma)$ signals that equilibrium profit may as well. While we know that P^B increases in $c(-\gamma)$, the effect of a change in c or γ on Π^* is unclear. As with $\widehat{q}$, apply the envelope theorem to attain

$$\frac{d\Pi}{dX} = \frac{P^*}{(\overline{q}^* - \underline{q}^*)^2}\left(-(\widehat{q} - \underline{q}^*)\frac{\partial \overline{q}^*}{\partial X} - (\overline{q}^* - \widehat{q})\frac{\partial \underline{q}^*}{\partial X}\right)$$

for $X \in \{c, \gamma\}$. Recall that if c alone increases, the interval $[\underline{q}, \overline{q}]$ contracts. How much each boundary moves, however, depends on their initial levels, cost, and price. Therefore, the sign of the above equation is ambiguous because the probability of sale could increase if $\widehat{q}$ becomes relatively closer to $\overline{q}$ than $\underline{q}$.

Figure 2.6: Profit as Search Costs Change



Profit compared to the limit. $c = .5$, $\gamma = 4$, $V_H = 5$, and $V_L = -5$.

The ambiguity of profit with respect to changes in search cost can also be seen as $c \to \infty$. As cost approaches infinity, the consumer never searches and price tends towards P^B. As this happens, the option value of search approaches 0, so that $P^B \to \widehat{q}V_H + (1 - \widehat{q})V_L$. If $\widehat{q}V_H + (1 - \widehat{q})V_L < 0$, then price and profit approach 0. If, however, $\widehat{q}V_H + (1 - \widehat{q})V_L > 0$, then whether this increase in cost is good or bad for the firm depends on how this price compares to the profit the firm was initially earning. Consider Figure 2.6. The only difference between the top and bottom panels is the value of $\widehat{q}$. The horizontal lines represent the initial expected value, or what profit converges to as $c \to \infty$. It is unclear whether increasing c will increase or decrease the firm's profits in the limit; for $\widehat{q} = .6$, the firm is better off with low costs, and for $\widehat{q} = .8$, the firm makes a higher profit with increased costs.

2.6 Product Design

Now consider a richer model in which the seller has control, not only over price, but also over some element of product design. Specifically, he is able to choose the dispersion of product value. For example, consider a firm that is choosing how innovative to be with a new product, such as a smart phone. If it chooses to be conservative and innovate very little relative to the last version of the phone, quality dispersion is low. In other words, the firm may not know if the new phone is of high or low quality, but the high-quality and low-quality products are very similar. If instead, the firm chooses to be very innovative and introduce many new features to the phone, the new product could be a great success or a great failure, and product quality dispersion is high.

To model this dispersion, I assume that the the seller chooses a mean-preserving spread of value, such that initial expected value, $\widehat{V} = \widehat{q}V_H + (1 - \widehat{q})V_L$, is fixed. This product design choice is another tool that the seller can use to encourage or discourage information acquisition by making it more or less appealing; the more dispersion there is, the more information the consumer can learn from product research, and the more enticing it becomes.

I discuss two ways to model this product quality dispersion.[22] The first approach is to fix the value of the low-type good, V_L, and allow the seller to choose V_H. The probability $\widehat{q}$ then adjusts so that $\widehat{V}$ remains fixed. Note that prior belief is determined by $\widehat{q} = (\widehat{V} - V_L)/(V_H - V_L)$ so that high choices of V_H are associated with low values of $\widehat{q}$

[22]Note that both approaches are special cases of the demand curve rotation discussed in Johnson and Myatt (2006).

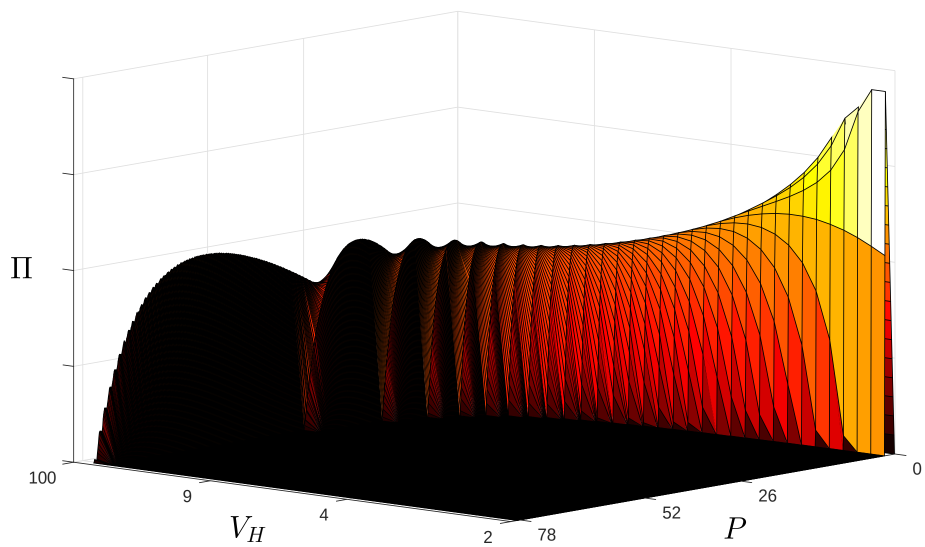
and visa versa. In other words, the seller can either choose to be very confident that the good is the high type if V_H is very close to initial expected value, or he can choose a much more unfavorable belief in exchange for the high-type good being very valuable. This is most easily interpreted when the buyer is viewed as a unit measure of consumers. Then increasing dispersion ($V_H \uparrow, \widehat{q} \downarrow$) indicates that fewer consumers are well-matched with the product, but those who are, obtain more value from it. In other words, with high dispersion, not many consumers like the product, but those that do like it, love it.

The second way to model dispersion is to fix the reputation of the firm, $\widehat{q}$, and again allow the seller to choose V_H. V_L then adjusts so that $\widehat{V}$ remains fixed. In this case, $V_L = (\widehat{V} - \widehat{q}V_L)/(1 - \widehat{q})$ so that high values of V_H are associated with low values of V_L. This scenario is most easily interpreted if the product is thought to have an underlying quality common to all consumers. Therefore, with the reputation of the firm constant, increasing dispersion ($V_H \uparrow$, $V_L \downarrow$) indicates that the high-quality object is more valuable, and the low-quality object is less valuable.

2.6.1 Immediate Sale

In previous work, Johnson and Myatt (2006) find that an extremal level of dispersion is always optimal. They consider consumers whose willingnesses to pay are drawn from a distribution. The monopolist has control over price and the dispersion of the distribution. They show that the firm desires either as little or as much dispersion as is allowed. In order to obtain this result, Johnson and Myatt first make a local argument; they fix the level of dispersion and show that if the optimal price is low (below some cutoff), then profit

Figure 2.7: Product Design Profit without Search



Profit with Extreme Level of Dispersion Optimal. $c = 1$, $\gamma = 4$, $\widehat{V} = 2$, and $V_L = -1$.

is decreasing in dispersion, while if optimal price is high, it is increasing in dispersion. They then make an additional assumption to make the local argument global.

I take a similar approach as Johnson and Myatt (2006) by analyzing how dispersion affects profits, given the pricing decision of the seller.

Proposition 2.9. *When it is optimal for the seller to discourage search, profit is decreasing in dispersion.*

Proof. See Appendix. □

Proposition 2.9 says that if the optimal price for the seller to post is P^B, profit is decreasing in V_H, regardless of how dispersion is modeled. Intuitively, higher dispersion increases the consumer's option value of search, lowering profit. To minimize this option value, the seller prefers to lower the level of dispersion by decreasing V_H. Therefore, if the seller's profit is maximized by selling the good immediately, he prefers the least amount of dispersion possible. An example of what profit might look like when immediate sale is better for the firm is shown in Figure 2.7. It is clear that the seller maximizes profit by reducing dispersion as much as possible ($V_H = \widehat{V}$) and posting a low price in order to sell the product immediately.

2.6.2 Dispersion and Search

Next, consider the case in which it is optimal for the seller to post a high price (P^*) and induce search. A sufficient condition for product research and some dispersion to be optimal is that selling immediately is infeasible ($P^B \leq 0$), even for the lowest level of dispersion. This will be true if $\widehat{V} \leq 0$ because $P^B \to \widehat{V}$ as $V_H \to \widehat{V}$.[23] In other words, if the initial expected value is too low, some dispersion is necessary to convince the consumer to gather information and give the firm a chance of selling the good. This is the case in which without consumer search, potentially beneficial trades are not made because they are initially too risky. Information acquisition allows the trade of these goods.

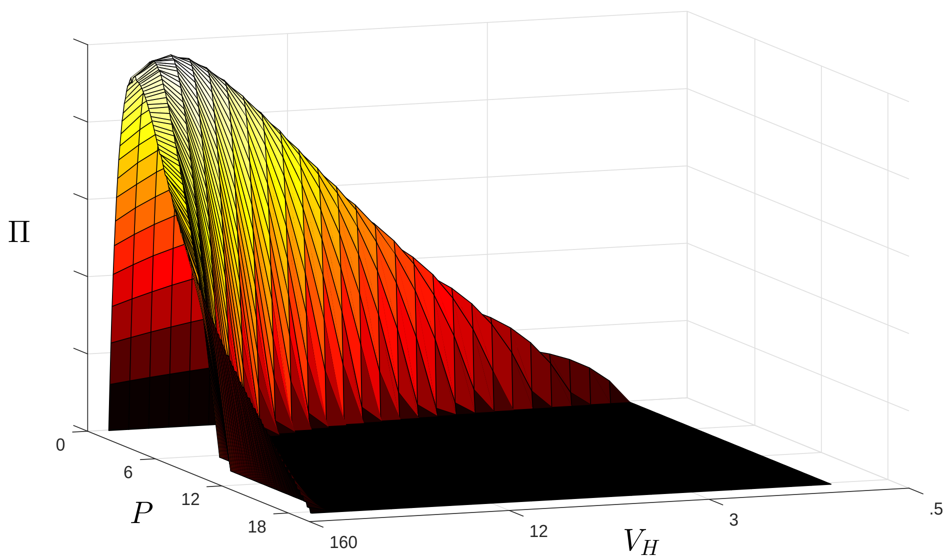
First, consider dispersion as a tradeoff between V_H and $\widehat{q}$. Then profit is

$$\Pi(P, V_H) = P\frac{\frac{\widehat{V}-V_L}{V_H-V_L} - \underline{q}(P, V_H)}{\overline{q}(P, V_H) - \underline{q}(P, V_H)},$$

which takes the reaction of the consumer to the joint choice of P and V_H into account. Invoking the envelope theorem shows how a change in V_H affects profit through both $\widehat{q}$ and

[23]Note that the same sufficient condition is obtained by allowing $V_H \to \widehat{V}$ in equation (2.10).

Figure 2.8: Product Design Profit with Search and Interior Dispersion



Profit with Interior Level of Dispersion Optimal. $c = .5$, $\gamma = .25$, $\widehat{V} = 0$, and $V_L = -5$.

the consumer's search strategy. Overall,

$$\frac{d\Pi}{dV_H} = \frac{P^*}{(\overline{q}^* - \underline{q}^*)^2}\left(\underbrace{-(\widehat{q} - \underline{q}^*)\frac{\partial \overline{q}^*}{\partial V_H} - (\overline{q}^* - \widehat{q})\frac{\partial \underline{q}^*}{\partial V_H}}_{V_H\uparrow}\underbrace{-\frac{(\overline{q}^* - \underline{q}^*)\widehat{q}^2}{\widehat{V} - V_L}}_{\widehat{q}\downarrow}\right). \tag{2.14}$$

Equation (2.14) shows that there are conflicting effects of increasing dispersion on equilibrium profit. On the one hand, V_H rising causes the range of beliefs for which a consumer will search, $[\underline{q}^*, \overline{q}^*]$, to shift down, increasing the probability of sale. This positively impacts profit, as seen in the first two terms of equation (2.14). On the other hand, the resulting fall in $\widehat{q}$ has a negative effect on profit by moving the belief relatively closer to the lower bound, as indicated by the third term in equation (2.14).

While the overall effect of an increase in dispersion is ambiguous, an interior level of dispersion may be optimal. Consider Figure 2.8. It is clear that the seller prefers to choose a high level of dispersion in order to raise the option value of search and induce the consumer to gather information. This dispersion comes at a cost, however, and too much of it deteriorates the probability of sale to such an extent that profit falls.

Some amount of dispersion is necessary for the consumer to do research, but Figure 2.8 shows that this level of dispersion need not be extremal. Intuitively, even though the positive effect in equation (2.14) ($[\underline{q}^*, \overline{q}^*]$ shifting down) dominates for small amounts of dispersion, as V_H becomes large and these bounds get closer to 0, they move less and less in response to additional changes. At some point, as V_H approaches infinity, the negative effect dominates, and profit begins to decrease in dispersion.

Next, consider dispersion as a tradeoff between V_H and V_L. In this case, the seller's profit is

$$\Pi(P, V_H) = P \frac{\widehat{q} - \underline{q}(P, V_H)}{\overline{q}(P, V_H) - \underline{q}(P, V_H)}. \tag{2.15}$$

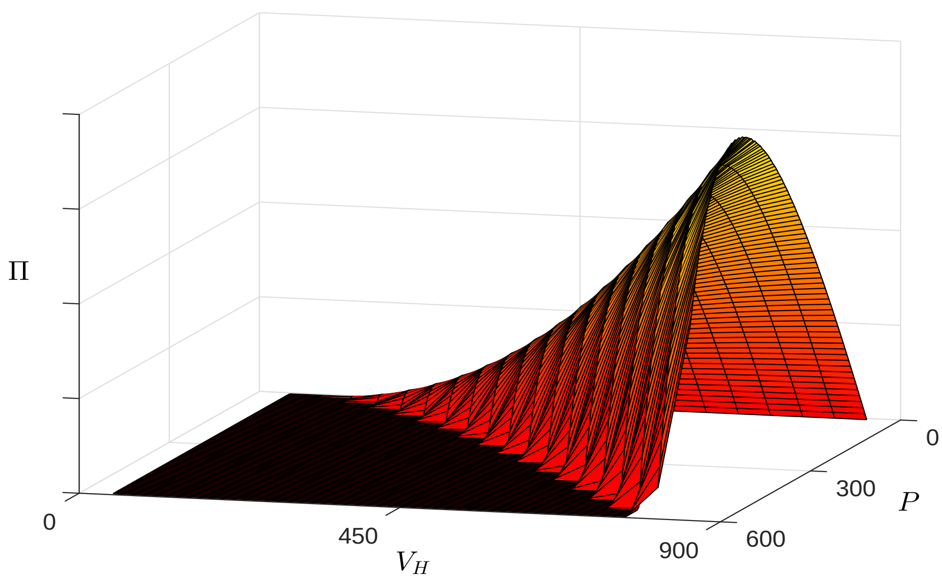
In order to understand how an increase in dispersion affects profit, the following lemma is needed.

Lemma 2.2. *Keeping the price fixed, an increase in dispersion, modeled as a tradeoff between V_H and V_L, causes $\underline{q}$ to decrease and $\overline{q}$ to increase, so that $[\underline{q}, \overline{q}]$ expands.*

Proof. See Appendix. □

Because a change in dispersion affects both V_H and V_L simultaneously, the consumer reacts to an increase in dispersion by adjusting her strategy in a different way than in Proposition 2.2. By expanding $[\underline{q}, \overline{q}]$, she does relatively more product research when dis-

Figure 2.9: Product Design Profit with Search and Extreme Dispersion



Profit with Extreme Level of Dispersion Optimal. $c = 5$, $\gamma = .0576$, $\widehat{V} = -10$, and $\widehat{q} = .8$.

persion increases. Again, invoking the envelope theorem, we see that the change in profit with respect to dispersion is

$$\frac{d\Pi}{dV_H} = \frac{P^*}{(\overline{q}^* - \underline{q}^*)^2}\left(\underbrace{-(\widehat{q} - \underline{q}^*)\frac{\partial \overline{q}^*}{\partial V_H}}_{V_L\downarrow}\underbrace{-(\overline{q}^* - \widehat{q})\frac{\partial \underline{q}^*}{\partial V_H}}_{V_H\uparrow}\right). \tag{2.16}$$

Equation (2.16) shows the conflicting effects of increasing dispersion on equilibrium profit. The resultant increase in $\overline{q}^*$ negatively impacts profit by making the upper bound more difficult to reach, lowering the probability of sale. The decrease in $\underline{q}^*$ positively affects profit, as it is now more unlikely that search will result in no sale.

To see what happens to profit as dispersion becomes large, note that the limit as

$V_H \to \infty$ of (2.16) is 0, and the limit of (2.15) is ∞.[24] This indicates that an extreme level of dispersion is optimal (i.e. profit is maximized as $V_H \to \infty$), and an example is shown in Figure 2.9.[25] If the level of dispersion is too low, the monopolist cannot sell the good at all, and profit is 0. As the level of dispersion is raised, however, profit increases as the high-type product becomes more and more valuable to the consumer.

Two factors influence the difference in outcomes between the two forms of dispersion. First, the tradeoff between V_H and $\widehat{q}$ is convex, while that of V_H and V_L is linear. This means that as dispersion approaches infinity, if the firm is decreasing match probability in favor of value, it concedes more and more probability each time it increases dispersion. If the firm trades off V_H for V_L, however, increasing dispersion involves the same reduction in value of the low-type good regardless of how much dispersion there is already. Second, as shown above, various ways of modeling dispersion affect the monopolist's profit differently. If modeled as a tradeoff between V_H and $\widehat{q}$, an increase in dispersion shifts $[\underline{q}^*, \overline{q}^*]$ down while simultaneously decreasing $\widehat{q}$ within that interval. If modeled as a tradeoff between V_H and V_L, however, an increase in dispersion expands $[\underline{q}^*, \overline{q}^*]$, symmetrically affecting the upper and lower bounds. In light of these differences, the most appropriate modeling choice depends on the characteristics of the specific market in question.

Overall, we see that an extreme choice of dispersion is optimal if the firm's objective is to sell the good right away but not necessarily if consumer search is optimal. If immediate

[24]This is because the limiting price, as defined by (2.8) in ∞ as the upper and lower bound approach 1 and 0 respectively.

[25]Note that this outcome is influenced by the lack of discounting in the model. In equilibrium, the consumer's upper bound nears 1, which will not be reached in finite time.

sale is best, the monopolist desires no dispersion because it increases the consumer's option value of search. If search is optimal, however, too much dispersion may eventually decrease the probability of sale, lowering profit.

2.6.3 Discussion

Consider the difference between these results and those in previous literature. In Johnson and Myatt, charging a high price means that the firm sells only to the small number of consumers who have very high valuations of the good. They show that if the firm prefers dispersion, raising the level of dispersion increases profit because the benefit of making the top portion of the distribution even more enthusiastic about the product outweighs the cost of selling a smaller quantity. My results indicate that in the presence of consumer search, this is not necessarily the case. When the firm desires search and therefore dispersion, an interior level may be optimal, as evidenced in Figure 2.8. The addition of information acquisition means that consumers are now strategic. The ways in which their strategies adjust to increases in dispersion therefore shape the profit function's response.

Wang (2016) adds a different kind of consumer search to a framework with a monopoly choice of dispersion. The author allows the firm to control the price and the informativeness of advertising, which operates similarly to dispersion. Consumers see advertisements for free, prior to costly search. Therefore, more informative advertising creates more dispersed expected values when consumers decide whether to search. If consumers do search, they pay a fixed cost for their match value to be revealed completely and immediately. Under an assumption on the demand curve that implies that the firm prefers consumers to remain uninformed and purchase the product immediately, Wang finds that an interior level of informativeness is optimal if the cost of search is low enough.

Finally, the analysis of consumer search with product design can be compared to Bar-Isaac et al. (2012), who find that firms with low initial values desire high levels of dispersion and those with high initial values prefer low levels of dispersion. Their model, which builds off of Johnson and Myatt's, considers many firms whose products each have an innate value and a consumer-specific component. Firms cannot alter their innate values, but they can choose the dispersion of the idiosyncratic quality. Just as in Johnson and Myatt's model, firms choose extreme levels of dispersion. Firms with low initial values use dispersion to compensate for their low values by selling only to buyers who are well-matched with their products. This is comparable to the sufficient condition for product research in my model because dispersion helps a firm with an initially low value to sell its product. In Bar-Isaac et al., firms use dispersion to sell to only the most well-matched consumers, and in the current model, the seller uses dispersion to incentivize search.

2.7 Conclusion

I conclude by discussing potential empirical implications and possible topics for future research.

2.7.1 Implications

It is always necessary in theoretical models to abstract away from reality to some extent for reasons of tractability and clarity. This does not mean, however, that the results obtained have no bearing on or insight into the real-world behavior of market participants.

For example, in the current model, one assumption that might be problematic for testing predictions of the model is that the seller is a monopolist.

The assumption that the firm is a monopolist is not as restrictive as it might initially appear, however, for two reasons. First, and most obviously, some markets may indeed be relatively close to monopolies if their products have very poor substitutes. Second, consider the case where there is some "standard" in the market, and the consumer wishes to find out about a new or less well-known good. In this case, as she learns about the new product, she always has the outside option to buy the "standard" good. This would be the same as walking away in my model where the value of the outside option is normalized to 0.

Adjusting our interpretation in this way yields three testable implications for online marketplaces, such as Amazon. First, consumers spend little or no time looking through comments and reviews, and purchase soon after arriving at the webpage if the posted price is low relative to the outside option. In other words, they will not find product research worthwhile if it is obvious they are being charged a low price.

Second, firms are more likely to post lower prices if their reputations are poor. If the value of the low-quality good is less than the marginal cost of production, the model predicts a smooth price increase for firms of higher and higher reputation because selling immediately is not an option for low beliefs. If the value is larger than marginal cost, however, which is arguably the more common case, my model predicts a discrete difference in the price charged by a firm aiming to sell immediately and one hoping to induce product research in equilibrium.

To see why and how there might be a discontinuity in price, consider websites like

Product Elf and AMZ Review Trader. They are a channel through which sellers contact interested Amazon consumers. In exchange for severely discounted products, consumers agree to write honest reviews about the good. If a seller's reputation is high enough, though, he has no need to resort to such drastic discounts, creating a discrete difference in price.

The third implication of my model is that the recent decline in search costs due to the availability of the internet will have had an overall ambiguous effect on posted prices. More specifically, goods that are priced to sell quickly will have seen price decreases, while goods with higher prices that induce consumer search may have seen price increases. This is because if the seller allows the consumer to gather additional information, it means he is confident that the signal the consumer receives will be positive on average. In order to keep incentives balanced after the cost of search falls, he may need to raise price to compensate.

2.7.2 Future Work

The model proposed in this paper constructs a very natural environment, so there are many more interesting questions that could be asked in a similar framework. For example, we have seen that whether the good has one value or many attributes is crucial to the seller's pricing decision. If the good has a single value, be it underlying or idiosyncratic, selling immediately is optimal when initial value is low, while the opposite is true if the good has many attributes. But the reality for many products lies somewhere in the middle. For example, some laptops are objectively better than others, breaking down less on average, while different features of a particular laptop (screen resolution, memory, etc.) may appeal more to one person than another. A model with both components could further identify optimal pricing strategies, depending on where a particular good falls on the spectrum.

It is also possible to consider multiple sellers, where the consumer can search across sellers and acquire information about only one good at a time. She has to form a strategy that decides not only when to purchase and walk away, but also when to switch from acquiring information about one good to gathering signals about the other. Ke et al. (2015) examine this problem for the many attributes case based off of Branco et al. (2012)'s model. Given how different the results in the current paper are from those in their baseline model, however, one could expect important differences in the equilibrium with multiple sellers as well.

Relatedly, one could consider multiple buyers where the first buyer gets a lower price than any subsequent consumers. Under these assumptions, the product is sold sooner than in the model with only one consumer, as buyers compete to receive the low price. How the seller's pricing strategy will be affected is not obvious. On the one hand, he has an incentive to raise prices relative to the current model to take advantage of the fact that consumers will be willing to buy the first good when less confident about quality in order to receive the low price. On the other hand, the seller does not want to raise prices too much, as it would deteriorate the probability of sale of the second item sold.

Finally, one could examine a different choice variable than price for the seller. As discussed above, firms can choose to sell their products for severely discounted prices in exchange for online reviews. It is also possible for them to pay companies to write favorable reviews of their products to increase their ratings. Either way, the seller is attempting to manipulate the signal consumers receive. This could be modeled as the seller choosing the variance of the signal process. It could also be seen as an upwards modification of the drift parameter, either once at the beginning of the game, or continuously, as a function of the current belief.

APPENDIX A
APPENDIX TO CHAPTER 1: OMITTED PROOFS

Proof of Proposition 1.1. As $1 - F_{-}(p_H) = \sigma_H$ and $1 - F_{-}(p_L) = \sigma_H + \sigma_L$, equation (1.3) reduces to

$$\frac{q^c}{1-q^c} = \frac{\widehat{q}}{1-\widehat{q}}\frac{\sigma_H + \sigma_L}{\sigma_H}.$$

In addition, because the low-type seller is indifferent between accepting and rejecting p_L,

$$r(p_L - c_L) = \lambda_L \sigma_H (c_H - p_L).$$

First, consider the equilibrium in which $p_L = v_L$. q^c is immediate from equation (1.4). Using this, σ_H and σ_L follow from the above two equations. This equilibrium exists if and only if $\sigma_L + \sigma_H \leq 1$, which is equivalent to condition (1.5).

Now consider the equilibrium in which $p_L < v_L$. In this case, no buyer has an incentive to make a losing offer, and thus $\sigma_L + \sigma_H = 1$. The four equilibrium variables in the proposition follow from the above two equations, equation (1.4) and this additional equation. This equilibrium exists if and only if $p_L = c_L + \widehat{q}\lambda_L(v_H - c_H)/(r(1-\widehat{q})) < v_L$, which is opposite to condition (1.5).

□

Proof of Proposition 1.2. We first characterize the case in which $p_L = v_L$. From buyers' indifference between c_H and v_L, $q^c = (c_H - v_L)/(v_H - v_L)$. The other three equilibrium conditions are

$$\frac{\sigma_H + \sigma_L}{\sigma_H} = \frac{1-\widehat{q}}{\widehat{q}}\frac{c_H - v_L}{v_H - c_H},\ r(v_L - c_L) = -\phi(\lambda_L) + \lambda_L \sigma_H (c_H - v_L),\ \phi'(\lambda_L) = \sigma_H(c_H - v_L).$$

From the last two conditions, $r(v_L - c_L) = -\phi(\lambda_L) + \lambda_L \phi'(\lambda_L)$, which implies $\lambda_L = \tilde{\lambda}$. Given λ_L, it follows that

$$\sigma_H = \frac{r(v_L - c_L) + \phi(\lambda_L)}{\lambda_L(c_H - v_L)}.$$

Finally, from the first condition,

$$\sigma_L = \left(\left(\frac{1-\widehat{q}}{\widehat{q}}\right)\frac{c_H - v_L}{v_H - c_H} - 1\right),\ \sigma_H = \left(\left(\frac{1-\widehat{q}}{\widehat{q}}\right)\frac{c_H - v_L}{v_H - c_H} - 1\right)\frac{r(v_L - c_L) + \phi(\lambda_L)}{\lambda_L(c_H - v_L)}.$$

This equilibrium exists if and only if $\sigma_H + \sigma_L \geq 1$, which is equivalent to condition (1.9).

Now we consider the case where $p_L < v_L$. In this case, no buyer has an incentive to offer a losing price, and thus $\sigma_H + \sigma_L = 1$. Now the four equilibrium conditions are

$$\begin{aligned} q^c &= \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})\sigma_H},\ \frac{1}{\sigma_H} = \frac{1-\widehat{q}}{\widehat{q}}\frac{c_H - p_L}{v_H - c_H}, \\ r(p_L - c_L) &= -\phi(\lambda_L) + \lambda_L \sigma_H (c_H - p_L),\ \phi'(\lambda_L) = \sigma_H (c_H - p_L). \end{aligned}$$

From the second and last conditions, $\phi'(\lambda_L) = \widehat{q}(v_H - c_H)/(1-\widehat{q})$. This identifies the unique value of λ_L. Then, it is straightforward to calculate the following:

$$p_L = c_L + \frac{\lambda_L \phi'(\lambda_L) - \phi(\lambda_L)}{r},\ \sigma_H = \left(\frac{\widehat{q}}{1-\widehat{q}}\right)\frac{v_H - c_H}{c_H - p_L},\ q^c = \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})\sigma_H}.$$

It suffices to show that the second equilibrium in which $p_L < v_L$ exists if and only if condition (1.9) fails. Note that

$$\begin{aligned} p_L < v_L &\Leftrightarrow r(p_L - c_L) = \lambda_L \phi'(\lambda_L) - \phi(\lambda_L) < r(v_L - c_L) \\ \Leftrightarrow \quad \phi'(\lambda_L) &< \frac{r(v_L - c_L) + \phi(\lambda_L)}{\lambda_L} \Leftrightarrow 1 < \left(\frac{1-\widehat{q}}{\widehat{q}}\right)\frac{r(v_L - c_L) + \phi(\lambda_L)}{\lambda_L(v_H - c_H)}. \end{aligned}$$

In other words, the equilibrium in which $p_L < v_L$ exists if this inequality holds. In order to show that this inequality is implied whenever (1.9) does not hold, define

$$H(\lambda) \equiv \left(\frac{1-\widehat{q}}{\widehat{q}}\right)\frac{r(v_L - c_L) + \phi(\lambda)}{\lambda(v_H - c_H)}.$$

By its definition, it suffices to prove that $H(\tilde{\lambda}) > 1$ implies $H(\lambda_L) > 1$. Notice that

$$H'(\lambda) = \left(\frac{1-\widehat{q}}{\widehat{q}}\right) \frac{\lambda\phi'(\lambda) - \phi(\lambda) - r(v_L - c_L)}{\lambda^2(v_H - c_H)}.$$

As $\lambda\phi'(\lambda) - \phi(\lambda)$ is strictly increasing and $H'(\tilde{\lambda}) = 0$, $H(\cdot)$ strictly decreases until $\tilde{\lambda}$ and then strictly increases. The result immediately follows from this property of $H(\cdot)$.

Now suppose condition (1.9) holds, but there exists an equilibrium in which $p_L < v_L$. From the equilibrium conditions,

$$\lambda_L\phi'(\lambda_L) - \phi(\lambda_L) = r(p_L - c_L) < r(v_L - c_L) = \tilde{\lambda}\phi'(\tilde{\lambda}).$$

As $\lambda\phi'(\lambda) - \phi(\lambda)$ is strictly increasing in λ, $\lambda_L < \tilde{\lambda}$. On the other hand, if (1.9) holds, then

$$\phi'(\tilde{\lambda}) = \frac{r(v_L - c_L) + \phi(\tilde{\lambda})}{\tilde{\lambda}} \leq \frac{\widehat{q}(v_H - c_H)}{1-\widehat{q}} = \phi'(\lambda_L).$$

The convexity of $\phi(\cdot)$ implies $\tilde{\lambda} < \lambda_L$, which is a contradiction to the previous conclusion.

□

Proof of Corollary 1.1. The first part is obvious, because $p_L = v_L$ as long as (1.10) holds. The second part follows from the explicit solution for p_L:

$$p_L = c_L + \frac{\lambda}{r}\frac{\widehat{q}(v_H - c_H)}{1-\widehat{q}} + \frac{1}{4rb}\left(\frac{\widehat{q}(v_H - c_H)}{1-\widehat{q}}\right)^2.$$

When b decreases, p_L increases, reflecting a benefit to the seller. Each buyer's expected payoff is equal to

$$\frac{(v_H - c_H)(v_L - p_L)}{v_H - p_L},$$

which increases in b. □

Proof of Proposition 1.3. We first consider the case where $\lambda_L > \lambda_H$. In this case, as explained in the main text, buyers' unconditional beliefs $q^u(t)$ are always increasing. Given this, it is straightforward to derive the following properties with standard arguments (see, e.g., Kim, 2015): buyers offer only $p_L(t)(< v_L)$ if $q^c(t) < q^* \equiv (c_H - v_L)/(v_H - v_L)$ and c_H if $q^c(t) > q^*$. If $q^c(t) = q^*$, then $p_L(t) = v_L$ and buyers are indifferent among c_H, $p_L(t)$, and a losing price.

- Let t_1^* be the point at which $q^c(t)$ reaches q^*. Before t_1^*, $q^c(0) < q^u(0) = \widehat{q} < q^*$, and thus buyers' beliefs evolve according to

$$\frac{q^u(t)}{1-q^u(t)} = \frac{\widehat{q}}{1-\widehat{q}}\frac{1}{e^{-\lambda_L t}}, \text{ and } \frac{q^c(t)}{1-q^c(t)} = \frac{\widehat{q}}{1-\widehat{q}}\frac{1}{e^{-\lambda_L t}}\frac{\lambda_H}{\lambda_L}, \text{ if } t < t_1^*.$$

  Therefore, the value of t_1^* can be found from the condition that

$$\frac{q^c(t_1^*)}{1-q^c(t_1^*)} = \frac{\widehat{q}}{1-\widehat{q}}\frac{1}{e^{-\lambda t_1^*}}\frac{\lambda_H}{\lambda_L} = \frac{c_H - v_L}{v_H - c_H}.$$

- Let t_2^* be the first time buyers offer c_H. Buyers are willing to offer c_H only when $q^c(t) \geq q^*$, whereas their beliefs strictly increase unless they offer only a losing price. Combining these with the above properties, it follows that buyers offer only c_H after t_2^*. The value of t_2^* is determined from the condition that

$$p_L(t_1^*) = c_L + e^{-r(t_2^*-t_1^*)}\frac{\lambda_L}{r+\lambda_L}(c_H - c_L) = v_L.$$

Now suppose $\lambda_L < \lambda_H$.[1] In this case, buyers' unconditional beliefs increase if trade occurs at $p_L(t)$ but decrease if trade occurs at c_H. Therefore, unlike in the previous case, it cannot be the case that buyers offer only c_H after a certain point and buyers' beliefs

[1] See Kim (2015) for the case where $\lambda_L = \lambda_H$.

converge to 1. As buyers' beliefs must be monotone in time (see, e.g., Kaya and Kim, 2015, for a formal argument), this means that there exits q^{**} to which $q^c(t)$ converges.

- Once $q^c(t)$ reaches q^*, it must stay constant. As it cannot be the case that buyers offer only a losing price, they must randomize between c_H and $p_L(t)$. For buyers' indifference between c_H and $p_L(t)$,

$$q^{**}(v_H - c_H) + (1 - q^{**})(v_L - c_H) = (1 - q^{**})(v_L - p_L(t)) \Leftrightarrow p_L(t) = p_L^* \equiv \frac{c_H - q^{**} v_H}{1 - q^{**}}.$$

For p_L^* to be the low-type seller's reservation price,

$$r(p_L^* - c_L) = \lambda_L \sigma_H(t)(c_H - p_L^*) \Leftrightarrow \sigma_H(t) = \sigma_H^* \equiv \frac{r(p_L^* - c_L)}{\lambda_L(c_H - p_L^*)}.$$

Finally, buyers' beliefs stay constant only when the two seller types exit the game at an identical rate, and thus

$$\lambda_H \sigma_H^* = \lambda_L(\sigma_L(t) + \sigma_H^*) \Leftrightarrow \sigma_L(t) = \sigma_L^* \equiv \frac{\lambda_H \sigma_H^*}{\lambda_L} - \sigma_H^*.$$

- There are two cases to consider, depending on whether $p_L^* = v_L$ or $p_L^* < v_L$.

  - $p_L^* = v_L$: applying the above equilibrium conditions,

  $$q^{**} = q^*,\ \sigma_H^* = \frac{r(v_L - c_L)}{\lambda_L(c_H - v_L)},\ \text{and } \sigma_H^* + \sigma_L^* = \frac{\lambda_H}{\lambda_L}\frac{r(v_L - c_L)}{\lambda_L(c_H - v_L)}.$$

  A necessary and sufficient condition for this to be an equilibrium is that

  $$\sigma_H^* + \sigma_L^* = \frac{\lambda_H}{\lambda_L}\frac{r(v_L - c_L)}{\lambda_L(c_H - v_L)} \leq 1.$$

  - $p_L^* < v_L$: in this case, buyers obtain a strictly positive expected payoff and, therefore, never offer a losing price, which yields another equilibrium condition $\sigma_H^* + \sigma_L^* = 1$. The four equilibrium variables, p_L^*, σ_H^*, σ_L^*, and q^{**}, can be obtained by combining the above three equilibrium conditions with this additional condition. It is straightforward to show that this equilibrium exists if and only if

$$\sigma_H^* + \sigma_L^* = 1 \leq \frac{\lambda_H}{\lambda_L} \frac{r(v_L - c_L)}{\lambda_L(c_H - v_L)}.$$

- Let t^* be the time at which $q^c(t)$ reaches q^{**}. There are two cases to consider, depending on whether $q^c(0) < q^{**}$ or not.

  – $q^c(0) < q^{**}$: this case arises when λ_H is relatively small:

  $$\frac{q^c(0)}{1 - q^c(0)} = \frac{\widehat{q}}{1 - \widehat{q}} \frac{\lambda_H}{\lambda_L} < \frac{q^{**}}{1 - q^{**}}.$$

  In this case, buyers offer only $p_L(t)$ until t^*, and thus

  $$\frac{q^c(t)}{1 - q^c(t)} = \frac{\widehat{q}}{1 - \widehat{q}} \frac{1}{e^{-\lambda_L t}} \frac{\lambda_H}{\lambda_L}, \text{ if } t < t^*.$$

  The value of t^* can be found from the condition that

  $$\frac{q^c(t^*)}{1 - q^c(t^*)} = \frac{\widehat{q}}{1 - \widehat{q}} \frac{1}{e^{-\lambda_L t^*}} \frac{\lambda_H}{\lambda_L} = \frac{q^{**}}{1 - q^{**}}.$$

  – $q^c(0) > q^{**}$: this case arises when λ_H is sufficiently large (when the above inequality is reversed). Buyers offer only c_H until t^*, and thus

  $$\frac{q^c(t)}{1 - q^c(t)} = \frac{\widehat{q}}{1 - \widehat{q}} \frac{e^{-\lambda_H t}}{e^{-\lambda_L t}} \frac{\lambda_H}{\lambda_L}, \text{ if } t < t^*.$$

  The value of t^* can be found from the condition that

  $$\frac{q^c(t^*)}{1 - q^c(t^*)} = \frac{\widehat{q}}{1 - \widehat{q}} \frac{e^{-\lambda_H t^*}}{e^{-\lambda_L t^*}} \frac{\lambda_H}{\lambda_L} = \frac{q^{**}}{1 - q^{**}}.$$

□

Proof of Proposition 1.5. The condition $r(v_L - c_L) < \underline{\lambda}(c_H - v_L)$ guarantees that the seller prefers receiving c_H from the next buyer (arriving at rate $\underline{\lambda}$) to accepting v_L immediately. Then, the case when $\overline{\lambda}$ is not available reduces to the non-stationary model in Kim (2015) (or, the limit case as $\overline{\lambda}$ tends to $\underline{\lambda}$ in Proposition 1.3). Let t^* be the value such that

$$\frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})e^{-\underline{\lambda}t^*}} = q^* = \frac{c_H - v_L}{v_H - v_L}.$$

In any equilibrium, buyers offer only $p_L(t)$ until t^*, from which the low-type seller's reservation price stays equal to v_L and buyers randomize between c_H and a losing price. The low-type seller's expected payoff at the beginning of the game is equal to $p_L(0) - c_L = e^{-rt^*}(v_L - c_L)$. Buyers before t^* obtain $(1 - q^u(t))(v_L - p_L(t))$ (where $q^u(t) = \widehat{q}/(\widehat{q} + (1-\widehat{q})e^{-\underline{\lambda}t})$), whereas those after t^* receive zero expected payoff.

In order to show that the low-type seller's expected payoff is lower when $\overline{\lambda}$ is available, notice that $q^u(t^*) = q^* < q^u(t_1^*)$ (that is, buyers' unconditional beliefs at t_1^* must exceed q^*), which implies that $t^* < t_1^*$. In addition, the low-type seller's reservation price at t_1^* falls short of v_L (i.e., $p_L(t_1^*) < v_L$). The result follows from the fact that the low-type seller's expected payoff when $\overline{\lambda}$ is available is equal to

$$p_L(0) - c_L = e^{-rt_1^*}(p_L(t_1^*) - c_L) < e^{-rt^*}(v_L - c_L).$$

For buyers' payoffs, first consider the buyers who arrive before $t_1^*(> t^*)$. As they offer only $p_L(t)$ and $q^u(t) = q^c(t)$ before t_1^*, their expected payoffs are also given by $(1 - q^u(t))(v_L - p_L(t))$. However, for the reason given above, $p_L(t)$ is lower, and thus their expected payoffs are higher, when $\overline{\lambda}$ is available. The payoff result for late buyers' payoffs is immediate from the fact that all buyers after $t^*(< t_1^* < t_3^*)$ receive 0 expected payoff when $\overline{\lambda}$ is not available, whereas all buyers after t_3^* obtain a strictly positive expected payoff when $\overline{\lambda}$ is available. □

APPENDIX B
APPENDIX TO CHAPTER 1: EQUILIBRIUM CONSTRUCTION IN THE NON-STATIONARY MODEL

This appendix constructs the unique equilibrium of the non-stationary model in Section 1.4. We omit an involved uniqueness proof. Interested readers are referred to our online appendix.

Last Phase

Consider the last phase where $t \geq t_3^*$. In this phase, buyers' unconditional beliefs $q^u(t)$ are large enough that they are willing to offer c_H even if the low-type seller chooses the high search intensity $\overline{\lambda}$ for sure. This provides a strong incentive for the low type to increase her search intensity and, therefore, she chooses the high search intensity $\overline{\lambda}$ with probability 1.

As all buyers offer c_H and the low type chooses $\overline{\lambda}$, her reservation price $p_L(t)$ is given by

$$r(p_L(t) - c_L) = -\phi + \overline{\lambda}(c_H - p_L(t)) \Leftrightarrow p_L(t) = \overline{p} \equiv \frac{-\phi + rc_L + \overline{\lambda}c_H}{r + \overline{\lambda}}.$$

As the low-type seller always chooses $\overline{\lambda}$, the relationship between buyers' unconditional beliefs and their conditional beliefs is given by

$$q^c(t) = \frac{q^u(t)\underline{\lambda}}{q^u(t)\underline{\lambda} + (1 - q^u(t))\overline{\lambda}} \Leftrightarrow \frac{q^u(t)}{1 - q^u(t)} = \frac{q^c(t)}{1 - q^c(t)} \frac{\overline{\lambda}}{\underline{\lambda}}. \quad \text{(B.1)}$$

Finally, because the high type trades at rate $\underline{\lambda}$, whereas the low-type seller trades at rate $\overline{\lambda}$, buyers' unconditional beliefs increase according to

$$q^u(t) = \frac{q^u(t_3^*)e^{-\underline{\lambda}(t-t_3^*)}}{q^u(t_3^*)e^{-\underline{\lambda}(t-t_3^*)} + (1-q^u(t_3^*))e^{-\overline{\lambda}(t-t_3^*)}} = \frac{q^u(t_3^*)}{q^u(t_3^*) + (1-q^u(t_3^*))e^{-(\overline{\lambda}-\underline{\lambda})(t-t_3^*)}}.$$

Observe that, because $\overline{\lambda} > \underline{\lambda}$, $q^u(t)$ is strictly increasing and converges to 1 as t increases.

It remains to identify the starting point of this phase (i.e., the initial condition for the above equations). It comes from the fact that t_3^* is at the intersection between the third and the last phases and, therefore, buyers must still be indifferent between c_H and a losing price:

$$q^c(t_3^*)(v_H - c_H) + (1-q^c(t_3^*))(v_L - c_H) = 0 \Leftrightarrow \frac{q^c(t_3^*)}{1-q^c(t_3^*)} = \frac{c_H - v_L}{v_H - c_H}.$$

Combining this equation with equation (B.1) leads to

$$\frac{q^u(t_3^*)}{1-q^u(t_3^*)} = \frac{c_H - v_L}{v_H - c_H}\frac{\overline{\lambda}}{\underline{\lambda}}. \tag{B.2}$$

Two Intermediate Phases

In the two intermediate phases, buyers' unconditional beliefs $q^u(t)$ are neither sufficiently large (they would be unwilling to offer c_H if the low-type seller chooses $\overline{\lambda}$) nor sufficiently small (buyers would be willing to offer c_H if the low-type seller chooses $\underline{\lambda}$). Therefore, in equilibrium, buyers must offer c_H and the low-type seller must choose $\overline{\lambda}$ with just right interior probabilities so that buyers are indifferent between c_H and $p_L(t)$ or a losing price, and the low-type seller remains indifferent between $\underline{\lambda}$ and $\overline{\lambda}$.

The Low-Type Seller's Reservation Prices and Buyers' Equilibrium Offer Strategies.

We first solve for the reservation price function $p_L(t)$ over the two intermediate phases. The low-type seller's indifference between $\underline{\lambda}$ and $\overline{\lambda}$ implies[1]

$$\begin{aligned} r(p_L(t) - c_L) &= -\phi + \overline{\lambda}\sigma_H(t)(c_H - p_L(t)) + \dot{p}_L(t) \\ &= \underline{\lambda}\sigma_H(t)(c_H - p_L(t)) + \dot{p}_L(t). \end{aligned}$$

Combining the two equations yields the following ordinary differential equation for $p_L(\cdot)$:

$$r(p_L(t) - c_L) = \frac{\phi\underline{\lambda}}{\overline{\lambda} - \underline{\lambda}} + \dot{p}_L(t).$$

This differential equation admits a closed-form solution:

$$p_L(t) = c_L + A + e^{r(t-t_1^*)}(p_L(t_1^*) - c_L - A), \qquad \text{(B.3)}$$

where $A \equiv \phi\underline{\lambda}/(r(\overline{\lambda} - \underline{\lambda}))$. Applying the solution back to the above equation,

$$\sigma_H(t) = \left(\frac{\phi}{\overline{\lambda} - \underline{\lambda}}\right) \frac{1}{c_H - c_L - A - e^{r(t-t_1^*)}(p_L(t_1^*) - c_L - A)}. \qquad \text{(B.4)}$$

It is clear that both $p_L(t)$ and $\sigma_H(t)$ strictly increase over time. Intuitively, buyers assign increasing probabilities to the high type and, therefore, offer the high price more frequently over time. This makes the low-type seller expect a higher payoff as well as exert more search effort.

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[1] As in Section 1.3, the equation can be obtained from the following recursive equation:

$$\begin{aligned} p_L(t) &= -\phi dt + \overline{\lambda}\sigma_H(t)c_H dt + e^{-(r+\overline{\lambda}\sigma_H(t))dt}p_L(t+dt) \\ &= \underline{\lambda}\sigma_H(t)c_H dt + e^{-(r+\underline{\lambda}\sigma_H(t))dt}p_L(t+dt). \end{aligned}$$

Unlike in Section 1.3, there is an additional term $\dot{p}_L(t)$ in the resulting equation. Mathematically, it is because of the difference between $p_L(t)$ and $p_L(t + dt)$. Economically, it is because the seller's search environment is no longer stationary. The term $\dot{p}_L(t)$ captures the effect of time passage on $p_L(t)$.

We now consider the evolution of beliefs. The following mathematical results, which, to our knowledge, have not been reported before, are useful.

Lemma B.1. *(1) If* $\xi(t) = \int_{\underline{t}}^{t} \underline{\lambda}\sigma_H(x)dx$ *where* $\sigma_H(t)$ *is given as in* (B.4)*, then*

$$\xi(t) = \frac{A}{c_H - c_L - A} \ln\left(\frac{(c_H - p_L(\underline{t}))e^{r(t-\underline{t})}}{(c_H - c_L - A) - e^{r(t-\underline{t})}(p_L(\underline{t}) - c_L - A)}\right).$$

(2) Suppose $q^u(t)$ *satisfies the ordinary differential equation of the form* $\dot{q}^u(t) = q^u(t)\left(Bq^u(t) - 1\right)\underline{\lambda}\sigma_H(t)$ *from time* $\underline{t}$*, where* B *is a fixed constant. The unique solution to the differential equation is given by*

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(\underline{t})} + B(e^{-\xi(t)} - 1)}.$$

Proof. $\xi(t)$ can be explicitly calculated as follows:

$$\begin{aligned}
\xi(t) &= \int_{\underline{t}}^{t} \frac{\phi\underline{\lambda}}{\overline{\lambda} - \underline{\lambda}} \frac{1}{(c_H - c_L - A) - e^{r(x-\underline{t})}(p_L(\underline{t}) - c_L - A)} dx \\
&= -A \int_{c_H - p_L(\underline{t})}^{(c_H - c_L - A) - e^{r(t-\underline{t})}(p_L(\underline{t}) - c_L - A)} \frac{1}{y((c_H - c_L - A) - y)} dy \\
&= -\frac{A}{c_H - c_L - A} \int_{c_H - p_L(\underline{t})}^{(c_H - c_L - A) - e^{r(t-\underline{t})(p_L(\underline{t}) - c_L - A)}} \left(\frac{1}{y} + \frac{1}{(c_H - c_L - A) - y}\right) dy \\
&= -\frac{A}{c_H - c_L - A} \ln\left(\frac{((c_H - c_L - A) - e^{r(t-\underline{t})}(p_L(\underline{t}) - c_L - A))(p_L(\underline{t}) - c_L - A)}{(c_H - p_L(\underline{t}))(e^{r(t-\underline{t})}(p_L(\underline{t}) - c_L - A))}\right) \\
&= \frac{A}{c_H - c_L - A} \ln\left(\frac{(c_H - p_L(\underline{t}))e^{r(t-\underline{t})}}{(c_H - c_L - A) - e^{r(t-\underline{t})}(p_L(\underline{t}) - c_L - A)}\right).
\end{aligned}$$

Let $\omega(t) = \ln(q^u(t)) + \xi(t)$. Then, the differential equation is equivalent to

$$\omega'(t) = Be^{\omega(t) - \xi(t)}\underline{\lambda}\sigma_H(t) \Leftrightarrow (-e^{-\omega(t)})' = B(-e^{-\xi(t)})'.$$

This implies that

$$e^{-\omega(t)} = e^{-\omega(\underline{t})} + B(e^{-\xi(t)} - e^{-\xi(t_2^*)}) = \frac{1}{q^u(\underline{t})} + B(e^{-\xi(t)} - 1).$$

Combining this with $q^u(t) = e^{\omega(t)-\xi(t)}$,

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(\underline{t})} + B(e^{-\xi(t)} - 1)}.$$

□

Third Phase Beliefs.

We first consider the third phase where $t \in [t_2^*, t_3^*)$. In this phase, buyers randomize between c_H and a losing price. This means that buyers' conditional beliefs $q^c(t)$ must be such that $q^c(t)(v_H - c_H) + (1 - q^c(t))(v_L - c_H) = 0$. Combining this equation with the general relationship between $q^u(t)$ and $q^c(t)$,

$$\frac{q^u(t)}{1 - q^u(t)} = \frac{q^c(t)}{1 - q^c(t)} \frac{\lambda_L(t)}{\underline{\lambda}} = \frac{c_H - v_L}{v_H - c_H} \frac{\lambda_L(t)}{\underline{\lambda}}. \tag{B.5}$$

In addition, as trade occurs only at c_H, by Bayes' rule,

$$q^u(t + dt) = \frac{q^u(t)e^{-\underline{\lambda}\sigma_H(t)dt}}{q^u(t)e^{-\underline{\lambda}\sigma_H(t)dt} + (1 - q^u(t))e^{-\lambda_L(t)\sigma_H(t)dt}},$$

which can be rewritten as

$$\dot{q}^u(t) = q^u(t)(1 - q^u(t))(\lambda_L(t) - \underline{\lambda})\sigma_H(t). \tag{B.6}$$

Combining equations (B.5) and (B.6) yields the following ordinary differential equation for $q^u(\cdot)$:

$$\dot{q}^u(t) = q^u(t) \left(\frac{q^u(t)}{q^*} - 1 \right) \underline{\lambda}\sigma_H(t).$$

Applying Lemma B.1, the solution is given by

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(t_2^*)} + \frac{e^{-\xi(t)}-1}{q^*}}. \tag{B.7}$$

Given $q^u(t)$, the low-type seller's search intensity $\lambda_L(t)$ can be explicitly derived from equation (B.5). Clearly, buyers' unconditional beliefs $q^u(t)$ and the low-type seller's optimal search intensity $\lambda_L(t)$ increase, whereas buyers' conditional beliefs $q^c(t)$ stay constant over the interval $[t_2^*, t_3^*]$.

Second Phase Beliefs.

Next, we study the second phase where $t \in [t_1^*, t_2^*)$. In this phase, buyers randomize between c_H and $p_L(t)$, and thus

$$q^c(t)(v_H - c_H) + (1 - q^c(t))(v_L - c_H) = (1 - q^c(t))(v_L - p_L(t)).$$

This means that buyers' unconditional beliefs $q^u(t)$ must be such that

$$\frac{q^u(t)}{1 - q^u(t)} = \frac{q^c(t)}{1 - q^c(t)} \frac{\lambda_L(t)}{\underline{\lambda}} = \frac{c_H - p_L(t)}{v_H - c_H} \frac{\lambda_L(t)}{\underline{\lambda}}. \tag{B.8}$$

In addition, because the low-type seller accepts not only c_H but also $p_L(t)$,

$$q^u(t + dt) = \frac{q^u(t) e^{-\underline{\lambda}\sigma_H(t)dt}}{q^u(t) e^{-\underline{\lambda}\sigma_H(t)dt} + (1 - q^u(t)) e^{-\lambda_L(t)dt}},$$

which implies that buyers' unconditional beliefs increase according to

$$\dot{q}^u(t) = q^u(t)(1 - q^u(t))(\lambda_L(t) - \underline{\lambda}\sigma_H(t)). \tag{B.9}$$

Similarly to the third phase, combining equations (B.8) and (B.9) and arranging the terms with the fact that $\sigma_H(t)(c_H - p_L(t)) = \phi/(\overline{\lambda} - \underline{\lambda})$ yield

$$\dot{q}^u(t) = q^u(t) \left(\left(\frac{\overline{\lambda} - \underline{\lambda}}{\phi} (v_H - c_H) + 1 \right) q^u(t) - 1 \right) \underline{\lambda}\sigma_H(t).$$

Again, applying Lemma B.1, the solution is given by

$$q^u(t) = \frac{e^{-\xi(t)}}{\frac{1}{q^u(t_1^*)} + \left(\frac{\overline{\lambda} - \underline{\lambda}}{\phi} (v_H - c_H) + 1 \right) (e^{-\xi(t)} - 1)}. \tag{B.10}$$

Given $p_L(t)$ and $q^u(t)$, buyers' conditional beliefs and the low-type seller's search intensity can be recovered from equation (B.8).

Notice that in the second phase, buyers' unconditional beliefs $q^u(t)$ are strictly increasing over time, whereas their conditional beliefs $q^c(\cdot)$ are strictly decreasing (see Figure 1.3). This is because $q^c(t) = (c_H - p_L(t))/(v_H - p_L(t))$ from (B.8), whereas $p_L(\cdot)$ is strictly increasing. This means that in the second phase the low-type seller's search intensity increases fast enough to more than offset the effect that increasing $q^u(t)$ has on $q^c(t)$.

Initial Phase

In the initial phase of the game, buyers assign a small probability to the high type. Therefore, buyers offer only $p_L(t)$, which induces the low-type seller not to increase her search intensity (i.e., $\lambda(t) = \underline{\lambda}$). This, in turn, implies that buyers' unconditional and conditional beliefs coincide, and both increase according to the baseline search intensity $\underline{\lambda}$:

$$q^u(t) = q^c(t) = \frac{\widehat{q}}{\widehat{q} + (1-\widehat{q})e^{-\underline{\lambda} t}}. \tag{B.11}$$

It is also clear that the low-type seller's reservation price increases according to

$$p_L(t) = c_L + e^{-r(t_1^* - t)}(p_L(t_1^*) - c_L).$$

Finding the Cutoff Time Points

We complete the equilibrium construction by finding the three cutoff time points, t_1^*, t_2^*, and t_3^*. We first determine the length of the third phase, $t_3^* - t_2^*$. From the equilibrium structure, $p_L(t_2^*) = v_L$ and $p_L(t_3^*) = \overline{p}$. The result then follows from the explicit solution of $p_L(\cdot)$ in (B.3):

$$e^{r(t_3^*-t_2^*)} = \frac{\overline{p} - c_L - A}{v_L - c_L - A}.$$

To solve for the other values, it is necessary to find $q^u(t_2^*)$. We use the fact that the value of $q^u(t_3^*)$ must be given as in equation (B.2). Combining this with equation (B.7) yields

$$\frac{(c_H - v_L)\overline{\lambda}}{(c_H - v_L)\overline{\lambda} + (v_H - c_H)\underline{\lambda}} = q^u(t_3^*) = \frac{e^{-\xi(t_3^*)}}{\frac{1}{q^u(t_2^*)} + \frac{e^{-\xi(t_3^*)}-1}{q^*}}.$$

The value of $q^u(t_2^*)$ can be obtained from this equation.

We now jointly determine $t_2^* - t_1^*$, $q^u(t_1^*)$, and $p_L(t_1^*)$, using the following facts. First, $q^u(t_2^*)$ from equation (B.10) must coincide with the value found above:

$$q^u(t_2^*) = \frac{e^{-\xi(t_2^*)}}{\frac{1}{q^u(t_1^*)} + \left(\frac{\overline{\lambda}-\underline{\lambda}}{\phi}(v_H - c_H) + 1\right)(e^{-\xi(t_2^*)} - 1)}. \tag{B.12}$$

Second, it must be that $\lambda_L(t_1^*) = \underline{\lambda}$: otherwise, buyers right before t_1^* would strictly prefer offering c_H to $p_L(t)$, because $p_L(\cdot)$ is always continuous, whereas $q^c(\cdot)$ would jump down at t_1^*. Applying this to equation (B.8) yields

$$\frac{1 - q^u(t_1^*)}{q^u(t_1^*)} = \frac{v_H - c_H}{c_H - p_L(t_1^*)}. \tag{B.13}$$

The existence of the solution follows from the fact that that in equation (B.13), the right-hand side is larger than the left-hand side if $t_2^* - t_1^*$ is sufficiently close to 0 (in which case $p_L(t_1^*)$ is close to v_L but $q^u(t_1^*)$ is not close to q^* by the solicitation curse), but the opposite is true if $t_2^* - t_1^*$ is sufficiently large (in which case $q^u(t_1^*)$ is close to 0 but the right-hand side is bounded above by $(v_H - c_H)/(c_H - v_L)$). The uniqueness follows from the fact that the right-hand side in equation (B.13) is strictly decreasing in $t_2^* - t_1^*$ (because $p_L(t_1^*)$

is strictly decreasing in $t_2^* - t_1^*$), whereas the left-hand side is strictly increasing in $t_2^* - t_1^*$: to show the latter, first notice that, as $p_L(t_2^*) = v_L$,

$$e^{-\xi(t_2^*)} = \left(\frac{c_H - p_L(t_1^*)}{c_H - v_L} \frac{v_L - c_L - A}{p_L(t_1^*) - c_L - A} \right)^{-\frac{A}{c_H - c_L - A}}.$$

Therefore, $e^{-\xi(t_2^*)}$ is strictly decreasing in $t_2^* - t_1^*$. Applying this to equation (B.12), it follows that $q^u(t_1^*)$ is also strictly decreasing in $t_2^* - t_1^*$.

Finally, we determine t_1^*. Given $q^u(t_1^*)$, this is immediate, because

$$q^u(t_1^*) = \frac{\widehat{q}}{\widehat{q} + (1 - \widehat{q})e^{-\lambda t_1^*}}.$$

APPENDIX C
APPENDIX TO CHAPTER 2

Proof of Proposition 2.2. We will use the implicit function theorem. Rearrange (2.4) to get

$$f_1(V_H, V_L, c, \gamma, \underline{q}, \overline{q}, P) = \overline{q}V_H + (1-\overline{q})V_L - P + \frac{2c}{\gamma}\left((1-2\overline{q})(\underline{\ln} - \overline{\ln}) - \frac{(1-2\underline{q})(\overline{q}-\underline{q})}{\underline{q}(1-\underline{q})}\right) \tag{C.1}$$

$$f_2(V_H, V_L, c, \gamma, \underline{q}, \overline{q}, P) = \underline{q}V_H + (1-\underline{q})V_L - P + \frac{2c}{\gamma}\left((1-2\underline{q})(\underline{\ln} - \overline{\ln}) - \frac{(1-2\overline{q})(\overline{q}-\underline{q})}{\overline{q}(1-\overline{q})}\right). \tag{C.2}$$

To apply the implicit function theorem, define the following:

$$\mu = (c, \gamma, V_H, V_L, P), q = (\underline{q}, \overline{q}), f = (f_1(q;\mu), f_2(q;\mu))$$

$$D_\mu f = \begin{bmatrix} \frac{\partial f_1}{\partial c} & \frac{\partial f_1}{\partial \gamma} & \frac{\partial f_1}{\partial V_H} & \frac{\partial f_1}{\partial V_L} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial c} & \frac{\partial f_2}{\partial \gamma} & \frac{\partial f_2}{\partial V_H} & \frac{\partial f_2}{\partial V_L} & \frac{\partial f_2}{\partial P} \end{bmatrix}$$

$$D_\mu \eta = \begin{bmatrix} \frac{d\underline{q}}{dc} & \frac{d\underline{q}}{d\gamma} & \frac{d\underline{q}}{dV_H} & \frac{d\underline{q}}{dV_L} & \frac{d\underline{q}}{dP} \\ \frac{d\overline{q}}{dc} & \frac{d\overline{q}}{d\gamma} & \frac{d\overline{q}}{dV_H} & \frac{d\overline{q}}{dV_L} & \frac{d\overline{q}}{dP} \end{bmatrix}$$

$$D_q f = \begin{bmatrix} \frac{\partial f_1}{\partial \underline{q}} & \frac{\partial f_1}{\partial \overline{q}} \\ \frac{\partial f_2}{\partial \underline{q}} & \frac{\partial f_2}{\partial \overline{q}} \end{bmatrix}$$

We find that

$$\frac{\partial f_2}{\partial \underline{q}} = \frac{\partial f_1}{\partial \overline{q}} = V_H - V_L + \frac{2c}{\gamma}\left(-2(\underline{\ln} - \overline{\ln}) - \frac{1-2\underline{q}}{\underline{q}(1-\underline{q})} + \frac{1-2\overline{q}}{\overline{q}(1-\overline{q})}\right) = 0,$$

so that

$$|D_q f| = \left(\frac{2c}{\gamma}\frac{\overline{q}-\underline{q}}{\overline{q}(1-\overline{q})\underline{q}(1-\underline{q})}\right)^2$$

and

$$-[D_q f]^{-1} = \frac{1}{|D_q f|}\begin{bmatrix} -\frac{\partial f_2}{\partial \overline{q}} & 0 \\ 0 & -\frac{\partial f_1}{\partial \underline{q}} \end{bmatrix} = \begin{bmatrix} -\frac{\gamma}{2c}\frac{\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}} & 0 \\ 0 & -\frac{\gamma}{2c}\frac{\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}} \end{bmatrix}$$

Comparative statics can be characterized by

$$D_\mu \eta = -[D_q f]^{-1} D_\mu f,$$

yielding

$$\begin{aligned}
\frac{d\overline{q}}{dc} &= -\frac{\overline{q}^2(1-\overline{q})^2}{c(\overline{q}-\underline{q})}\left((1-2\underline{q})(\underline{\mathrm{ln}}-\overline{\mathrm{ln}}) + \frac{(2\overline{q}-1)(\overline{q}-\underline{q})}{\overline{q}(1-\overline{q})}\right) \\
&= -\frac{\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}}\frac{\gamma}{2c^2}(P-\underline{q}V_H-(1-\underline{q})V_L) < 0 \\
\frac{d\underline{q}}{dc} &= \frac{\underline{q}^2(1-\underline{q})^2}{c(\overline{q}-\underline{q})}\left((2\overline{q}-1)(\underline{\mathrm{ln}}-\overline{\mathrm{ln}}) + \frac{(1-2\underline{q})(\overline{q}-\underline{q})}{\underline{q}(1-\underline{q})}\right) \\
&= \frac{\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}}\frac{\gamma}{2c^2}(\overline{q}V_H+(1-\overline{q})V_L-P) > 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{q}}{d\gamma} &= \frac{\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}}\frac{1}{\gamma}\left((1-2\underline{q})(\underline{\mathrm{ln}}-\overline{\mathrm{ln}}) + \frac{(2\overline{q}-1)(\overline{q}-\underline{q})}{\overline{q}(1-\overline{q})}\right) \\
&= \frac{\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}}\frac{1}{2c}(P-\underline{q}V_H-(1-\underline{q})V_L) > 0 \\
\frac{d\underline{q}}{d\gamma} &= -\frac{\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}}\frac{1}{\gamma}\left((2\overline{q}-1)(\underline{\mathrm{ln}}-\overline{\mathrm{ln}}) + \frac{(1-2\underline{q})(\overline{q}-\underline{q})}{\underline{q}(1-\underline{q})}\right) \\
&= -\frac{\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}}\frac{1}{2c}(\overline{q}V_H+(1-\overline{q})V_L-P) < 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\overline{q}}{dV_L} &= -\frac{\gamma}{2c}\frac{(1-\underline{q})\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}} < 0 \\
\frac{d\underline{q}}{dV_L} &= -\frac{\gamma}{2c}\frac{(1-\overline{q})\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}} < 0
\end{aligned}$$

$$\frac{d\overline{q}}{dV_H} = -\frac{\gamma}{2c}\frac{\underline{q}\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}} < 0$$
$$\frac{d\underline{q}}{dV_H} = -\frac{\gamma}{2c}\frac{\overline{q}\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}} < 0$$

$$\frac{d\overline{q}}{dP} = \frac{\gamma}{2c}\frac{\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}} > 0$$
$$\frac{d\underline{q}}{dP} = \frac{\gamma}{2c}\frac{\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}} > 0$$

□

Proof of Lemma 2.1. We will calculate the probability of sale directly from the signal, X_t, then convert it to the belief space. This is possible because there is a one to one relationship between the signal and beliefs. Recall that

$$dX_t = \alpha dt + \sigma dZ_t$$
$$q = \frac{1}{1 + (1-\widehat{q})/\widehat{q}\exp\left(\frac{-2\mu X}{\sigma^2}\right)}$$
$$\exp\left(-\frac{2\mu}{\sigma^2}\overline{X}\right) = \frac{\widehat{q}}{1-\widehat{q}}\frac{1-\overline{q}}{\overline{q}}$$
$$\exp\left(-\frac{2\mu}{\sigma^2}\underline{X}\right) = \frac{\widehat{q}}{1-\widehat{q}}\frac{1-\underline{q}}{\underline{q}},$$

where $\overline{X}$ is the cumulative signal at which the consumer purchases, and $\underline{X}$ is the cumulative signal at which the consumer walks away. For $a > 0$, $b > 0$, the probability that a process with a drift α and variance σ^2 increases a before decreasing b is

$$\frac{1 - e^{(2\alpha/\sigma^2)b}}{e^{(-2\alpha/\sigma^2)a} - e^{(2\alpha/\sigma^2)b}}.$$

Therefore,

$$\begin{aligned}
\text{Pr(sale)} &= \text{Pr}(X \uparrow \overline{X} \text{ before } X \downarrow \underline{X}) \\
&= \frac{1-\exp\left(-\frac{2\alpha}{\sigma^2}\underline{X}\right)}{\exp\left(-\frac{2\alpha}{\sigma^2}\overline{X}\right)-\exp\left(-\frac{2\alpha}{\sigma^2}\underline{X}\right)} \\
&= \widehat{q}(\text{Pr}\,|\alpha=\mu)+(1-\widehat{q})(\text{Pr}\,|\alpha=-\mu) \\
&= \widehat{q}\left(\frac{1-\exp\left(-\frac{2\mu}{\sigma^2}\underline{X}\right)}{\exp\left(-\frac{2\mu}{\sigma^2}\overline{X}\right)-\exp\left(-\frac{2\mu}{\sigma^2}\underline{X}\right)}\right)+(1-\widehat{q})\left(\frac{1-\exp\left(\frac{2\mu}{\sigma^2}\underline{X}\right)}{\exp\left(\frac{2\mu}{\sigma^2}\overline{X}\right)-\exp\left(\frac{2\mu}{\sigma^2}\underline{X}\right)}\right) \\
&= \widehat{q}\left(\frac{1-\frac{\widehat{q}}{1-\widehat{q}}\frac{1-\underline{q}}{\underline{q}}}{\frac{\widehat{q}}{1-\widehat{q}}\frac{1-\overline{q}}{\overline{q}}-\frac{\widehat{q}}{1-\widehat{q}}\frac{1-\underline{q}}{\underline{q}}}\right)+(1-\widehat{q})\left(\frac{1-\frac{1-\widehat{q}}{\widehat{q}}\frac{\underline{q}}{1-\underline{q}}}{\frac{1-\widehat{q}}{\widehat{q}}\frac{\overline{q}}{1-\overline{q}}-\frac{1-\widehat{q}}{\widehat{q}}\frac{\underline{q}}{1-\underline{q}}}\right) \\
&= \frac{\widehat{q}-\underline{q}(P)}{\overline{q}(P)-\underline{q}(P)}
\end{aligned}$$

□

Proof of Proposition 2.3. In order to do comparative statics on P^B, we must first do them on $\underline{q}^B$ via the implicit function theorem. Define

$$f = \frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)} + 2\underline{\ln}^B - \frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})} - 2\widehat{\ln} - \frac{\gamma}{2c}(V_H - V_L)$$

$$D_\mu f = \begin{bmatrix} \frac{\partial f}{\partial c} & \frac{\partial f}{\partial \gamma} & \frac{\partial f}{\partial \widehat{q}} \end{bmatrix}$$

$$D_\mu \eta = \begin{bmatrix} \frac{d\underline{q}^B}{dc} & \frac{d\underline{q}^B}{d\gamma} & \frac{d\underline{q}^B}{d\widehat{q}} \end{bmatrix}$$

$$-[D_q f]^{-1} = -\frac{1}{\frac{\partial f}{\partial \underline{q}^B}}, \qquad D_\mu \eta = -[D_q f]^{-1} D_\mu f,$$

Then we have

$$\frac{\partial f}{\partial \underline{q}^B} = -\frac{1}{(\underline{q}^B)^2(1-\underline{q}^B)^2},$$

so that

$$\begin{aligned}
\frac{d\underline{q}^B}{dc} &= \frac{\gamma}{2c^2}(V_H - V_L)(\underline{q}^B)^2(1-\underline{q}^B)^2 > 0 \\
\frac{d\underline{q}^B}{d\gamma} &= -\frac{1}{2c}(V_H - V_L)(\underline{q}^B)^2(1-\underline{q}^B)^2 < 0 \\
\frac{d\underline{q}^B}{d\widehat{q}} &= \frac{(\underline{q}^B)^2(1-\underline{q}^B)^2}{\widehat{q}^2(1-\widehat{q})^2} > 0.
\end{aligned}$$

That easily gives us the marginal changes in P^B.

$$\begin{aligned}
\frac{dP^B}{dc} &= \frac{2}{\gamma}\left((1-2\underline{q}^B)\left(\underline{\ln}^B - \widehat{\ln}\right) - \frac{(1-2\widehat{q})(\widehat{q}-\underline{q}^B)}{\widehat{q}(1-\widehat{q})}\right) \\
&= \frac{1}{c}(P - \underline{q}V_H - (1-\underline{q})V_L) > 0 \\
\frac{dP^B}{d\gamma} &= -\frac{c}{\gamma}\frac{dP^B}{dc} < 0 \\
\frac{dP^B}{d\widehat{q}} &= \frac{2c}{\gamma}\frac{\widehat{q}-\underline{q}^B}{\widehat{q}^2(1-\widehat{q})^2} > 0
\end{aligned} \tag{C.3}$$

□

Proof of Proposition 2.4. Let us start by examining what happens as $\widehat{q} \to 0$ (so $\underline{q}^B \to 0$).

First note

$$\lim_{\widehat{q}\to 0} \frac{\underline{q}^B}{\widehat{q}} = \lim_{\widehat{q}\to 0} \frac{d\underline{q}^B}{d\widehat{q}} = \lim_{\widehat{q}\to 0} \left(\frac{\underline{q}^B(1-\underline{q}^B)}{\widehat{q}(1-\widehat{q})}\right)^2 = \left(\lim_{\widehat{q}\to 0} \frac{\underline{q}^B}{\widehat{q}}\right)^2 \in \{0, 1, \infty\}.$$

In addition, it must be the case that $d\text{Pr(sale)}/dP(P^B) \to -\infty$ as $\widehat{q} \to 0$. In other words, allowing search will almost surely result in no sale, indicating an infinitely steep drop from

Pr(sale)$(P^B) = 1$. Therefore,

$$\lim_{\widehat{q}\to 0} -\frac{\widehat{q}^2(1-\widehat{q})^2}{(\widehat{q}-\underline{q}^B)^2} = \lim_{\widehat{q}\to 0} -\left(\frac{\widehat{q}}{\widehat{q}-\underline{q}^B}\right)^2 = -\left(\lim_{\widehat{q}\to 0}\frac{1}{1-\frac{d\underline{q}^B}{d\widehat{q}}}\right)^2 = -\left(\lim_{\widehat{q}\to 0}\frac{1}{1-\left(\frac{\underline{q}^B}{\widehat{q}}\right)^2}\right)^2 = -\infty,$$

so that $\lim_{\widehat{q}\to 0}\underline{q}^B/\widehat{q} = 1$. To find $\lim_{\widehat{q}\to 0} P^B$, note

$$\lim_{\widehat{q}\to 0}(1-2\widehat{q})(\underline{\ln}^B - \widehat{\ln}) = \ln\left(\lim_{\widehat{q}\to 0}\frac{\widehat{q}}{\underline{q}^B}\right) = 0$$

$$\lim_{\widehat{q}\to 0}\frac{(\widehat{q}-\underline{q}^B)(1-2\underline{q}^B)}{\underline{q}^B(1-\underline{q}^B)} = \lim_{\widehat{q}\to 0}\frac{\widehat{q}-\underline{q}^B}{\underline{q}^B} = \lim_{\widehat{q}\to 0}\frac{1-\frac{d\underline{q}^B}{d\widehat{q}}}{\frac{d\underline{q}^B}{d\widehat{q}}} = \lim_{\widehat{q}\to 0}\left(\left(\frac{\widehat{q}}{\underline{q}^B}\right)^2 - 1\right) = 0,$$

so that $\lim_{\widehat{q}\to 0} P^B = V_L$. Therefore,

$$\lim_{\widehat{q}\to 0} 1 - P^B\left(\frac{\widehat{q}(1-\widehat{q})}{\widehat{q}-\underline{q}^B}\right)^2 = \begin{cases} -\infty & \text{if } V_L > 0 \\ \infty & \text{if } V_L < 0. \end{cases}$$

Next consider $\widehat{q}\to 1$ (so $\underline{q}^B \to 1$). First note

$$\lim_{\widehat{q}\to 1}\frac{1-\underline{q}^B}{1-\widehat{q}} = \lim_{\widehat{q}\to 1}\frac{d\underline{q}^B}{d\widehat{q}} = \left(\lim_{\widehat{q}\to 1}\frac{1-\underline{q}^B}{1-\widehat{q}}\right)^2 \in \{0, 1, \infty\}.$$

It must be that dPr(sale)$/dP(P^B) \to 0$ as $\widehat{q}\to 1$ because searching will almost surely result in a sale. Therefore,

$$\lim_{\widehat{q}\to 1} -\left(\frac{\widehat{q}^2(1-\widehat{q})^2}{(\widehat{q}-\underline{q}^B)^2}\right) = -\left(\lim_{\widehat{q}\to 1}\frac{1-\widehat{q}}{\widehat{q}-\underline{q}^B}\right)^2 = -\left(\lim_{\widehat{q}\to 1}\frac{-1}{1-\frac{d\underline{q}^B}{d\widehat{q}}}\right)^2 = -\left(\lim_{\widehat{q}\to 1}\frac{-1}{1-\left(\frac{1-\underline{q}^B}{1-\widehat{q}}\right)^2}\right)^2 = 0,$$

so that $\lim_{\widehat{q}\to 1}(1-\underline{q}^B)/(1-\widehat{q}) = \infty$. To find $\lim_{\widehat{q}\to 1} P^B$, note

$$\lim_{\widehat{q}\to 1}(1-2\widehat{q})(\underline{\ln}^B - \widehat{\ln}) = -\lim_{\widehat{q}\to 1}\ln\left(\frac{1-\underline{q}^B}{1-\widehat{q}}\right) = -\infty$$

$$\lim_{\widehat{q}\to 1} -\frac{(\widehat{q}-\underline{q}^B)(1-2\underline{q}^B)}{\underline{q}^B(1-\underline{q}^B)} = \lim_{\widehat{q}\to 1}\frac{\widehat{q}-\underline{q}^B}{1-\underline{q}^B} = \lim_{\widehat{q}\to 1}\frac{1-\frac{d\underline{q}^B}{d\widehat{q}}}{-\frac{d\underline{q}^B}{d\widehat{q}}} = \lim_{\widehat{q}\to 1}\left(-\left(\frac{1-\widehat{q}}{1-\underline{q}^B}\right)^2 + 1\right) = 1$$

so that $\lim_{\widehat{q}\to 1} P^B = -\infty$. Finally, using equation (C.3),

$$\begin{aligned}
\lim_{\widehat{q}\to 1} 1 - P^B\left(\frac{\widehat{q}(1-\widehat{q})}{\widehat{q}-\underline{q}^B}\right)^2 &= 1 - \lim_{\widehat{q}\to 1} \frac{P^B}{\left(\frac{\widehat{q}-\underline{q}^B}{\widehat{q}(1-\widehat{q})}\right)^2} \\
&= 1 - \lim_{\widehat{q}\to 1} \frac{\frac{dP^B}{d\widehat{q}}}{2\frac{\widehat{q}-\underline{q}^B}{(1-\widehat{q})^3}\left((1-\widehat{q})\left(1-\frac{d\underline{q}^B}{d\widehat{q}}\right)+\widehat{q}-\underline{q}^B\right)} \\
&= 1 - \lim_{\widehat{q}\to 1} \frac{\frac{2c}{\gamma}\frac{\widehat{q}-\underline{q}^B}{(1-\widehat{q})^2}}{2\frac{\widehat{q}-\underline{q}^B}{(1-\widehat{q})^3}\left((1-\widehat{q})\left(1-\left(\frac{1-\underline{q}^B}{1-\widehat{q}}\right)^2\right)+\widehat{q}-\underline{q}^B\right)} \\
&= 1 - \lim_{\widehat{q}\to 1} \frac{c}{\gamma}\frac{1}{\frac{1-\underline{q}^B}{1-\widehat{q}}\left(1-\frac{1-\underline{q}^B}{1-\widehat{q}}\right)} = 1 > 0
\end{aligned}$$

□

Proof of Proposition 2.5. We can easily see that the left-hand side of (2.10) is decreasing in c:

$$\frac{dFOC(P^B)}{dc} = -\frac{dP^B}{dc}\left(\frac{\widehat{q}(1-\widehat{q})}{\widehat{q}-\underline{q}^B}\right)^2 - 2P^B(\widehat{q}(1-\widehat{q}))^2(\widehat{q}-\underline{q}^B)^{-3}\frac{d\underline{q}^B}{dc} < 0.$$

We must then examine the limits as c approaches 0 and ∞. As $c \to 0$, $\underline{q}^B \to 0$. Then to see what happens to P^B, consider

$$\begin{aligned}
\lim_{c\to 0}(1-2\widehat{q})(\underline{\ln}^B - \widehat{\ln})\frac{2c}{\gamma} &= (1-2\widehat{q})\frac{2}{\gamma}\lim_{c\to 0} c\underline{\ln}^B \\
&= (1-2\widehat{q})\frac{2}{\gamma}\lim_{c\to 0}\frac{\underline{\ln}^B}{1/c} \\
&= (1-2\widehat{q})\frac{2}{\gamma}\lim_{c\to 0}\frac{-\frac{1}{\underline{q}^B(1-\underline{q}^B)}\frac{d\underline{q}^B}{dc}}{-1/c^2} \\
&= (1-2\widehat{q})(V_H - V_L)\lim_{c\to 0}\underline{q}^B(1-\underline{q}^B) \\
&= 0.
\end{aligned}$$

$$\lim_{c\to 0} -\frac{2c}{\gamma}\frac{(\widehat{q}-\underline{q}^B)(1-2\underline{q}^B)}{\underline{q}^B(1-\underline{q}^B)} = -\frac{2}{\gamma}\widehat{q}\lim_{c\to 0}\frac{c}{\underline{q}^B(1-\underline{q}^B)}$$
$$= -\frac{2}{\gamma}\widehat{q}\lim_{c\to 0}\frac{1}{(1-2\underline{q}^B)\frac{d\underline{q}^B}{dc}}$$
$$= -\left(\frac{2}{\gamma}\right)^2\widehat{q}\frac{1}{V_H-V_L}\lim_{c\to 0}\left(\frac{c}{\underline{q}^B(1-\underline{q}^B)}\right)^2$$

This means $\lim_{c\to 0}\frac{c}{\underline{q}^B(1-\underline{q}^B)} \in \{0, \frac{\gamma}{2}(V_H-V_L)\}$. But if the limit was 0, then $\lim_{c\to 0} P^B = \widehat{q}V_H + (1-\widehat{q})V_L$, which would contradict the definition of P^B being the price at which the consumer is indifferent between buying and searching. Therefore, $\lim_{c\to 0}\frac{c}{\underline{q}^B(1-\underline{q}^B)} = \frac{\gamma}{2}(V_H-V_L)$, so that $\lim_{c\to 0} -\frac{2c}{\gamma}\frac{(\widehat{q}-\underline{q}^B)(1-2\underline{q}^B)}{\underline{q}^B(1-\underline{q}^B)} = -\widehat{q}(V_H-V_L)$. Overall, $\lim_{c\to 0} P^B = V_L$, so that $\lim_{c\to 0} FOC(P^B) = 1 - V_L(1-\widehat{q})^2$. The sufficient condition will always hold for $V_L \leq 0$, and for $V_L > 0$ if $\widehat{q} > 1 - V_L^{-1/2}$.

Next, as $c \to \infty$, $\underline{q}^B \to \widehat{q}$. To see what happens to P^B, consider

$$\lim_{c\to\infty}\frac{2c}{\gamma}(1-2\widehat{q})(\underline{\ln}^B - \widehat{\ln}) = \frac{2}{\gamma}(1-2\widehat{q})\lim_{c\to\infty}\frac{\ln\left(\frac{1-\underline{q}^B}{\underline{q}^B}\frac{\widehat{q}}{1-\widehat{q}}\right)}{\frac{1}{c}}$$
$$= (1-2\widehat{q})\frac{2}{\gamma}\lim_{c\to\infty}\frac{-\frac{1}{\underline{q}^B(1-\underline{q}^B)}\frac{d\underline{q}^B}{dc}}{-1/c^2}$$
$$= (1-2\widehat{q})(V_H-V_L)\lim_{c\to\infty}\underline{q}^B(1-\underline{q}^B)$$
$$= (1-2\widehat{q})(V_H-V_L)\widehat{q}(1-\widehat{q}).$$

$$\lim_{c\to\infty} -\frac{2c}{\gamma}\frac{(\widehat{q}-\underline{q}^B)(1-2\underline{q}^B)}{\underline{q}^B(1-\underline{q}^B)} = -\frac{2}{\gamma}\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}\lim_{c\to\infty}\frac{\widehat{q}-\underline{q}^B}{1/c}$$
$$= -\frac{2}{\gamma}\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}\lim_{c\to\infty}\frac{-\frac{d\underline{q}^B}{dc}}{-\frac{1}{c^2}}$$
$$= -(1-2\widehat{q})(V_H-V_L)\widehat{q}(1-\widehat{q})$$

Therefore, $\lim_{c\to\infty} P^B = \widehat{q}V_H + (1-\widehat{q})V_L$. Then $\lim_{c\to\infty} FOC(P^B) = -\infty$. Then (2.10) never holds, regardless of other parameters. Because FOC(P^B) is decreasing in c, this implies the existence of c' and c''. A symmetric argument can be made for γ. □

Proof of Proposition 2.7. $\overline{q}^*$, $\underline{q}^*$, and P^* will be determined by the system of (C.1), (C.2), and

$$f_3(V_H, V_L, c, \gamma, \underline{q}, \overline{q}, P) = \frac{\widehat{q} - \underline{q}^*}{\overline{q}^* - \underline{q}^*} - P^* \frac{\gamma}{2c} \frac{(\overline{q}^* - \widehat{q})(\underline{q}^*)^2(1-\underline{q}^*)^2 + (\widehat{q} - \underline{q}^*)(\overline{q}^*)^2(1-\overline{q}^*)^2}{(\overline{q}^* - \underline{q}^*)^3}. \tag{C.4}$$

To apply the implicit function theorem,

$$D_q f = \begin{bmatrix} \frac{\partial f_1}{\partial \underline{q}} & 0 & -1 \\ 0 & \frac{\partial f_2}{\partial \overline{q}} & -1 \\ \frac{\partial f_3}{\partial \underline{q}} & \frac{\partial f_3}{\partial \overline{q}} & \frac{\partial f_3}{\partial P} \end{bmatrix}$$

$$|D_q f| = \frac{\partial f_1}{\partial \underline{q}} \frac{\partial f_2}{\partial \overline{q}} \left(\frac{\partial f_3}{\partial P} + \frac{\partial f_3}{\partial \overline{q}} \frac{\partial \overline{q}}{\partial P} + \frac{\partial f_3}{\partial \underline{q}} \frac{\partial \underline{q}}{\partial P} \right). \tag{C.5}$$

The term in parentheses in equation (C.5) is the second-order condition (SOC) of the profit function, and will therefore be negative at any interior solution.

$$-[D_q f]^{-1} = -\frac{1}{\frac{\partial f_1}{\partial \underline{q}} \frac{\partial f_2}{\partial \overline{q}} SOC} \begin{bmatrix} \frac{\partial f_2}{\partial \overline{q}} \frac{\partial f_3}{\partial P} + \frac{\partial f_3}{\partial \overline{q}} & -\frac{\partial f_3}{\partial \overline{q}} & \frac{\partial f_2}{\partial \overline{q}} \\ -\frac{\partial f_3}{\partial \underline{q}} & \frac{\partial f_1}{\partial \underline{q}} \frac{\partial f_3}{\partial P} + \frac{\partial f_3}{\partial \underline{q}} & \frac{\partial f_1}{\partial \underline{q}} \\ -\frac{\partial f_2}{\partial \overline{q}} \frac{\partial f_3}{\partial \underline{q}} & -\frac{\partial f_1}{\partial \underline{q}} \frac{\partial f_3}{\partial \overline{q}} & \frac{\partial f_1}{\partial \underline{q}} \frac{\partial f_2}{\partial \overline{q}} \end{bmatrix}$$

Then with

$$D_\mu f = \begin{bmatrix} \frac{\partial f_1}{\partial \widehat{q}} \\ \frac{\partial f_2}{\partial \widehat{q}} \\ \frac{\partial f_3}{\partial \widehat{q}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{\partial f_3}{\partial \widehat{q}} \end{bmatrix}, D_\eta f = \begin{bmatrix} \frac{d\underline{q}^*}{d\widehat{q}} \\ \frac{d\overline{q}^*}{d\widehat{q}} \\ \frac{dP^*}{d\widehat{q}} \end{bmatrix}$$

$$\begin{aligned} \frac{dP^*}{d\widehat{q}} &= -\frac{1}{SOC} \left(\frac{1}{\overline{q} - \underline{q}} - \frac{P}{(\overline{q} - \underline{q})^2} \left(-\frac{d\underline{q}}{dP} + \frac{d\overline{q}}{dP} \right) \right) \\ &= -\frac{1}{SOC(\overline{q} - \underline{q})} \left[1 - \frac{(\widehat{q} - \underline{q})(\overline{q}^2(1-\overline{q})^2 - \underline{q}^2(1-\underline{q})^2)}{(\widehat{q} - \underline{q})\overline{q}^2(1-\overline{q})^2 + (\overline{q} - \widehat{q})\underline{q}^2(1-\underline{q})^2} \right] > 0, \end{aligned}$$

so at an interior maximum, an increase in prior belief always increases price. □

Proof of Proposition 2.8. Using the same strategy as in Proposition 2.7, the marginal effects of the parameters are given by

$$\begin{bmatrix} \frac{d\underline{q}^*}{dc} & \frac{d\underline{q}^*}{d\gamma} \\ \frac{d\overline{q}^*}{dc} & \frac{d\overline{q}^*}{d\gamma} \\ \frac{dP^*}{dc} & \frac{dP^*}{d\gamma} \end{bmatrix} = -\frac{1}{\frac{\partial f_1}{\partial \underline{q}}\frac{\partial f_2}{\partial \overline{q}} SOC} \begin{bmatrix} \frac{\partial f_2}{\partial \overline{q}}\frac{\partial f_3}{\partial P} + \frac{\partial f_3}{\partial \overline{q}} & -\frac{\partial f_3}{\partial \overline{q}} & \frac{\partial f_2}{\partial \overline{q}} \\ -\frac{\partial f_3}{\partial \underline{q}} & \frac{\partial f_1}{\partial \underline{q}}\frac{\partial f_3}{\partial P} + \frac{\partial f_3}{\partial \underline{q}} & \frac{\partial f_1}{\partial \underline{q}} \\ -\frac{\partial f_2}{\partial \overline{q}}\frac{\partial f_3}{\partial \underline{q}} & -\frac{\partial f_1}{\partial \underline{q}}\frac{\partial f_3}{\partial \overline{q}} & \frac{\partial f_1}{\partial \underline{q}}\frac{\partial f_2}{\partial \overline{q}} \end{bmatrix} \begin{bmatrix} \frac{\partial f_1}{\partial c} & \frac{\partial f_1}{\partial \gamma} \\ \frac{\partial f_2}{\partial c} & \frac{\partial f_2}{\partial \gamma} \\ \frac{\partial f_3}{\partial c} & \frac{\partial f_3}{\partial \gamma} \end{bmatrix}$$

Therefore, for $X \in \{c, \gamma\}$,

$$\frac{dP^*}{dX} = -\frac{1}{SOC}\left(\frac{\partial \underline{q}}{\partial X}\frac{\partial f_3}{\partial \underline{q}} + \frac{\partial \overline{q}}{\partial X}\frac{\partial f_3}{\partial \overline{q}} + \frac{\partial f_3}{\partial X}\right),$$

where $\frac{\partial \underline{q}}{\partial X}$ and $\frac{\partial \overline{q}}{\partial X}$ are given in the proof of Proposition 2.2, and

$$\frac{\partial f_3}{\partial c} = \frac{1}{c}\frac{\widehat{q} - \underline{q}}{\overline{q} - \underline{q}}$$

$$\frac{\partial f_3}{\partial \gamma} = -\frac{1}{\gamma}\frac{\widehat{q} - \underline{q}}{\overline{q} - \underline{q}}$$

$$\frac{\partial f_3}{\partial \overline{q}} = \frac{(\widehat{q} - \underline{q})}{(\overline{q} - \underline{q})^2\left((\widehat{q} - \underline{q})\frac{\partial \overline{q}}{\partial P} + (\overline{q} - \widehat{q})\frac{\partial \underline{q}}{\partial P}\right)}\left(2\frac{\partial \overline{q}}{\partial P}(\widehat{q} - \underline{q}) - \frac{\partial \underline{q}}{\partial P}(2\widehat{q} - \underline{q} - \overline{q}) - \frac{\gamma}{c}(\widehat{q} - \underline{q})\overline{q}(1 - \overline{q})(1 - 2\overline{q})\right)$$

$$\frac{\partial f_3}{\partial \underline{q}} = \frac{-1}{(\overline{q} - \underline{q})^2\left((\widehat{q} - \underline{q})\frac{\partial \overline{q}}{\partial P} + (\overline{q} - \widehat{q})\frac{\partial \underline{q}}{\partial P}\right)}\left(2\frac{\partial \overline{q}}{\partial P}(\widehat{q} - \underline{q})^2 + \frac{\partial \underline{q}}{\partial P}(\overline{q} - \widehat{q})(\overline{q} + 2\widehat{q} - 3\underline{q}) + \frac{\gamma}{c}(\widehat{q} - \underline{q})(\overline{q} - \widehat{q})\underline{q}(1 - \underline{q})(1 - 2\underline{q})\right).$$

Therefore,

$$h(\overline{q}, \underline{q}, \widehat{q}, \gamma, c) = \left(2\frac{d\overline{q}}{dP}(\widehat{q} - \underline{q})^2 + \frac{d\underline{q}}{dP}(\overline{q} - \widehat{q})(\overline{q} + 2\widehat{q} - 3\underline{q}) + \frac{\gamma}{c}(\widehat{q} - \underline{q})(\overline{q} - \widehat{q})\underline{q}(1 - \underline{q})(1 - 2\underline{q})\right)$$

$$g(\overline{q}, \underline{q}, \widehat{q}, \gamma, c) = (\widehat{q} - \underline{q})\left(\frac{d\underline{q}}{dP}(2\widehat{q} - \underline{q} - \overline{q}) - 2\frac{d\overline{q}}{dP}(\widehat{q} - \underline{q}) + \frac{\gamma}{c}(\widehat{q} - \underline{q})\overline{q}(1 - \overline{q})(1 - 2\overline{q})\right).$$

□

Proof of Proposition 2.9. First consider the form of dispersion in which there is a tradeoff between V_H and $\widehat{q}$. It is equivalent to consider a change in $\widehat{q}$ where $V_H = \frac{1}{\widehat{q}}(\widehat{V} - (1 - \widehat{q})V_L)$. Then

$$P^B = \widehat{V} - \frac{2c}{\gamma}\left(-(1 - 2\widehat{q})(\underline{\ln}^B - \widehat{\ln}) + \frac{(\widehat{q} - \underline{q}^B)(1 - 2\underline{q}^B)}{\underline{q}^B(1 - \underline{q}^B)}\right), \quad \text{(C.6)}$$

where $\underline{q}^B$ is defined by

$$\frac{2c}{\gamma}\left(\frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)}+2\underline{\ln}^B\right)=\frac{2c}{\gamma}\left(\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}+2\widehat{\ln}\right)+\frac{1}{\widehat{q}}(\widehat{V}-V_L).$$

First consider how $\underline{q}^B$ changes in $\widehat{q}$. We have

$$\begin{aligned}
f &= \frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)}+2\underline{\ln}^B-\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}-2\widehat{\ln}-\frac{\gamma}{2c}\frac{1}{\widehat{q}}(\widehat{V}-V_L)\\
\frac{\partial f}{\partial \underline{q}^B} &= -\frac{1}{(\underline{q}^B)^2(1-\underline{q}^B)^2}\\
\frac{\partial f}{\partial \widehat{q}} &= \frac{1}{\widehat{q}^2(1-\widehat{q})^2}\left(4\widehat{q}(1-\widehat{q})+(1-2\widehat{q})^2+\frac{\gamma}{2c}(\widehat{V}-V_L)(1-\widehat{q})^2\right),
\end{aligned}$$

so that

$$\frac{d\underline{q}^B}{d\widehat{q}}=-\frac{\frac{\partial f}{\partial \widehat{q}}}{\frac{\partial f}{\partial \underline{q}^B}}=\left(\frac{\underline{q}^B(1-\underline{q}^B)}{\widehat{q}(1-\widehat{q})}\right)^2\left(1+\frac{\gamma}{2c}(\widehat{V}-V_L)(1-\widehat{q})^2\right).$$

Plugging this and the fact that $f=0$ into P^B yields

$$\frac{dP^B}{d\widehat{q}}=-\frac{2c}{\gamma}\left(\frac{\underline{q}^B}{\widehat{q}}\left(2(\underline{\ln}^B-\widehat{\ln})-\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}+\frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)}\right)-\frac{\widehat{q}-\underline{q}^B}{\widehat{q}^2(1-\widehat{q})^2}\right)\geq 0.$$

Next consider dispersion where there is a tradeoff between V_H and V_L. P^B is still determined by (C.6), and $\underline{q}^B$ is defined by

$$\frac{2c}{\gamma}\left(\frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)}+2\underline{\ln}^B\right)=\frac{2c}{\gamma}\left(\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}+2\widehat{\ln}\right)+\frac{1}{1-\widehat{q}}(V_H-\widehat{V}).$$

Then

$$\begin{aligned}
f &= \frac{1-2\underline{q}^B}{\underline{q}^B(1-\underline{q}^B)}+2\underline{\ln}^B-\frac{1-2\widehat{q}}{\widehat{q}(1-\widehat{q})}-2\widehat{\ln}-\frac{\gamma}{2c}\frac{1}{1-\widehat{q}}(V_H-\widehat{V})\\
\frac{\partial f}{\partial \underline{q}^B} &= -\frac{1}{(\underline{q}^B)^2(1-\underline{q}^B)^2}\\
\frac{\partial f}{\partial V_H} &= -\frac{\gamma}{2c}\frac{1}{1-\widehat{q}},
\end{aligned}$$

so that

$$\frac{d\underline{q}^B}{dV_H} = -\frac{\frac{\partial f}{\partial V_H}}{\frac{\partial f}{\partial \underline{q}^B}} = -\frac{\gamma}{2c}\frac{1}{1-\widehat{q}}(\underline{q}^B)^2(1-\underline{q}^B)^2.$$

Plugging this P^B yields

$$\frac{dP^B}{dV_H} = -\frac{\widehat{q}-\underline{q}^B}{1-\widehat{q}} < 0.$$

□

Proof of Lemma 2.2. The proof follows the same form as that of Proposition 2.2. Now

$$f_1(V_H,\widehat{q},c,\gamma,\underline{q},\overline{q},P) = \frac{(\overline{q}-\widehat{q})V_H + (1-\overline{q})\widehat{V}}{1-\widehat{q}} - P + \frac{2c}{\gamma}\left((1-2\overline{q})(\underline{\ln}-\overline{\ln}) - \frac{(1-2\underline{q})(\overline{q}-\underline{q})}{\underline{q}(1-\underline{q})}\right)$$
$$f_2(V_H,\widehat{q},c,\gamma,\underline{q},\overline{q},P) = \frac{-(\widehat{q}-\underline{q})V_H + (1-\underline{q})\widehat{V}}{1-\widehat{q}} - P + \frac{2c}{\gamma}\left((1-2\underline{q})(\underline{\ln}-\overline{\ln}) - \frac{(1-2\overline{q})(\overline{q}-\underline{q})}{\overline{q}(1-\overline{q})}\right).$$

It is still true that $\partial f_2/\partial \underline{q} = \partial f_1/\partial \overline{q} = 0$ so that $-[D_q f]^{-1}$ is as it is in Proposition 2.2.

Therefore,

$$\frac{d\overline{q}}{dV_H} = \frac{\gamma}{2c}\frac{\overline{q}^2(1-\overline{q})^2}{\overline{q}-\underline{q}}\frac{\widehat{q}-\underline{q}}{1-\widehat{q}} > 0$$
$$\frac{d\underline{q}}{dV_H} = -\frac{\gamma}{2c}\frac{\underline{q}^2(1-\underline{q})^2}{\overline{q}-\underline{q}}\frac{\overline{q}-\widehat{q}}{1-\widehat{q}} < 0.$$

□

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